Event Shape and Determination of α_s at LEP

Outline

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- Hadronic Events at LEP
- Event Shape Distributions
- Determination of α_s
- Summary

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Motivation

➠ Event shape variables are sensitive to the rate of hard gluon emission $\longrightarrow \alpha_{\mathrm{s}}$

Thrust: $T \; = \; \left(\sum_a |\vec{p}_a \cdot \hat{n}_T| / \sum_a |\vec{p}_a|\right)_{\max}$

Heavy jet mass: $\rho_{\mathrm{H}} \; = \; \left(\sum_{a \in S_\pm} p_a / \sqrt{\mathrm{s}}\right)^2_{\max}$

Jet broadenings: compute in each hemisphere the quantity

$$\mathrm{B}_\pm \; = \; \frac{\sum_{i \in \mathrm{S}_\pm} |\vec{\mathrm{p}}_\mathrm{i} \times \vec{\mathrm{n}}_\mathrm{T}|}{2 \sum_i |\vec{\mathrm{p}}_\mathrm{i}|}.$$

Then $\mathrm{B_T} = \mathrm{B}_+ + \mathrm{B}_-$ and $\mathrm{B_W} = \max(\mathrm{B}_+, \mathrm{B}_-)$

C Parameter: from eigenvalues of linearised momentum tensor

$$\theta^{ij} = \frac{\sum_a \mathrm{p}_a^i \mathrm{p}_a^j / |\vec{\mathrm{p}}_a|}{\sum_a |\vec{\mathrm{p}}_a|} \quad i, j = 1, 2, 3 \; ; \Rightarrow \mathrm{C} \; = \; 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1)$$

Jet resolution parameter: $\mathrm{y}_3 \equiv$ value of jet resolution parameter in $\mathrm{k}_\perp$ algorithm at which event changes from 2-jet to 3-jet configuration

➠ Precise theoretical calculations exist for these variables:

Fixed order calculation exists up to $\mathcal{O}(\alpha_s{}^2)$ for all shape variables. Cumulative cross section $R(y)$ for the event shape variable y:

$$R(y,\alpha_s) \equiv \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy}$$
$$= \overline{\alpha}_s A(y) + \overline{\alpha}_s^2 \left[B(y) + 2\pi\beta_0 \ln\left(\frac{Q^2}{s}\right) A(y)\right]$$

with $\overline{\alpha}_s = \frac{\alpha_s(Q)}{2\pi}$; $\beta_0 = \frac{33-2N_F}{12\pi}$

$A(y)$, $B(y)$ computed by integrating ERT matrix elements

Describes data well in the multi-jet region, but fails in the two jet region (small y)

Resummation of Large Logarithms: For certain variables ($y \equiv 1 - \mathrm{T}$, ρ_H, $\mathrm{B_T}$, $\mathrm{B_W}$, $\frac{\mathrm{C}}{6}$, $\mathrm{y_3}$) leading and next to leading log terms are summed to all orders in

$$L \equiv \ln\left(\frac{1}{x_L y}\right)$$

	LL	NLL	Sub-leading			
$\mathcal{O}(\alpha_\mathrm{s})$	$\overline{\alpha}_\mathrm{s} L^2$	$\overline{\alpha}_\mathrm{s} L$	$\overline{\alpha}_\mathrm{s}$	$\overline{\alpha}_\mathrm{s} \mathcal{O}\left(\frac{1}{L}\right)$		
$\mathcal{O}(\alpha_\mathrm{s}{}^2)$	$\overline{\alpha}_\mathrm{s}^2 L^3$	$\overline{\alpha}_\mathrm{s}^2 L^2$	$\overline{\alpha}_\mathrm{s}^2 L$	$\overline{\alpha}_\mathrm{s}^2$	$\overline{\alpha}_\mathrm{s}^2 \mathcal{O}\left(\frac{1}{L}\right)$	
$\mathcal{O}(\alpha_\mathrm{s}{}^3)$	$\overline{\alpha}_\mathrm{s}^3 L^4$	$\overline{\alpha}_\mathrm{s}^3 L^3$	$\overline{\alpha}_\mathrm{s}^3 L^2$	$\overline{\alpha}_\mathrm{s}^3 L$	$\overline{\alpha}_\mathrm{s}^3$	$\overline{\alpha}_\mathrm{s}^3 \mathcal{O}\left(\frac{1}{L}\right)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

- Resummed LL and NLL terms dominate at small y
- Sub-leading terms important at large y

$\Rightarrow$ Combine fixed order calculations with resummed ones

Avoid double counting of terms present in $(\alpha_s{}^2)$ fixed order and resummed calculations $\Rightarrow$ Matching Schemes

Log R Matching Take log of fixed order; expand in power series and match in $\ln R(y)$

R Matching Remove $\mathcal{O}\ (\alpha_s{}^2)$ terms from resummed $R(y)$; take fixed order calculation in toto

Also use kinematic constraints :Cross sections must vanish beyond kinematic limit

$$R\Big|_{y=y_{\max}} = 1; \quad \frac{dR}{dy}\Big|_{y=y_{\max}} = 0.$$

Modified Log R Replace L in the resummed terms by $L' = \ln\left((\frac{1}{x_L \cdot y})^p - (\frac{1}{x_L \cdot y_{\max}})^p + 1\right)$ with modification degree $p \geq 1$; carry out Log R matching

Modified R L is modified and the matching coefficients become functions of y to match the kinematic condition

- All matching algorithms exact up to $\mathcal{O}\left(\alpha_s{}^2\right)$

To include hadronisation effects convolute:

$$f(y) \quad = \quad \int f^{\text{pert}}(y') \cdot p^{\text{non-pert}}(y', y)\, dy'$$

where $p^{\text{non-pert}}(y', y)$ is the probability to find a value y after fragmentation and decays for a parton level value y' evaluated using Parton Shower Monte Carlo programs

Advantage of this approach:

- Can explain small y (high statistics) region
- Get good fit at a reasonable scale ($Q \approx \sqrt{s}$)
- Uncertainty due to uncalculated terms are reduced

α_s measured with good precision

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- All 4 LEP experiments measured these event shape variables at different beam energies
- Using radiative events at LEP I, event shape distributions are measured at reduced centre-of-mass energies

α_s determined over a large energy region

- Extraction of α_s by the 4 experiments is discussed in the LEP QCD working group

Homogeneous approach among the 4 sets of measurements

Hadronic Events

Characterised by

- large visible energy
- high multiplicity
- transverse energy balance

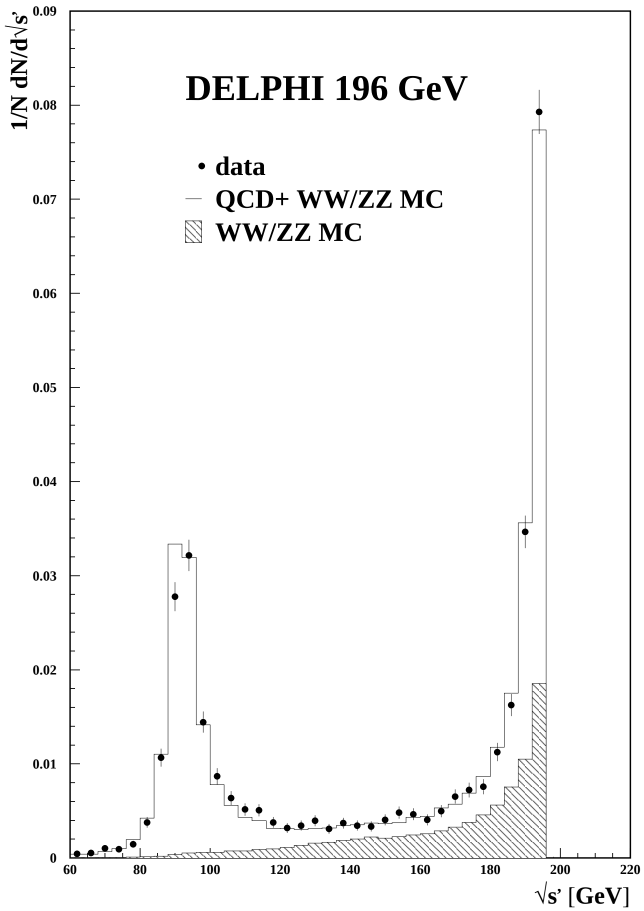
LEP provides decent $\mathcal{L}$ at high energies

Typical sample size:

$\sqrt{s}$ (GeV)	$\int \mathcal{L}\cdot dt$ (pb^{-1})	ϵ (%)	Π (%)	#
130.1	6.1	90.0	80.6	556
136.1	5.9	89.0	81.5	414
161.3	10.8	89.0	81.2	424
172.3	10.2	84.8	82.6	325
182.8	55.3	84.2	82.4	1500
188.6	176.8	87.8	81.1	4479
194.4	112.2	82.8	81.4	2403
200.2	117.0	85.7	80.6	2456
206.2	207.6	86.0	78.8	4146

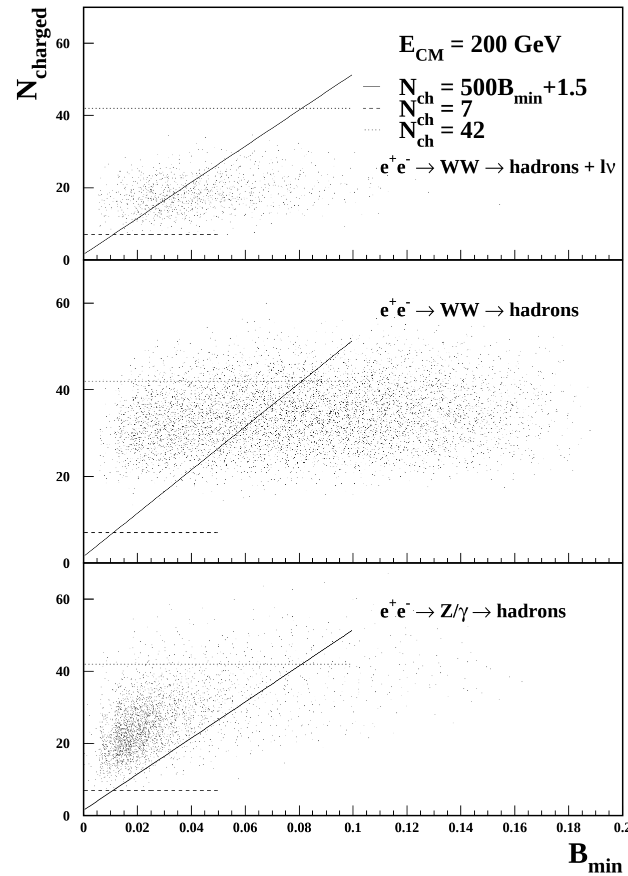
Large background at high energies due to

Radiative return to Z



$\Rightarrow$ Cut on $\sqrt{s'}$

Four fermion processes



$\Rightarrow$ Active rejection of 4-jet events

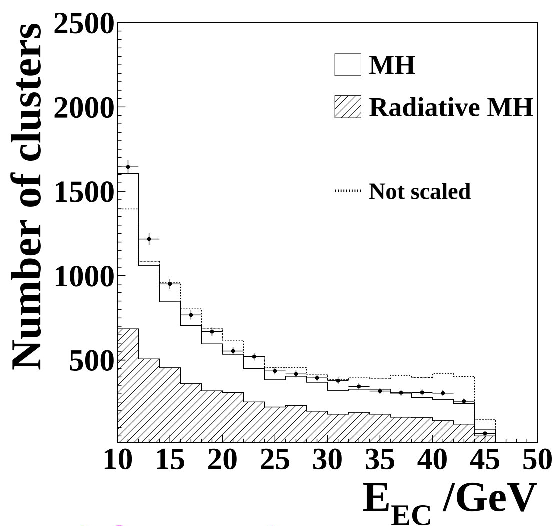
❒ Select hadronic events with standard cuts (OPAL)

❒ Find at least one high energy isolated photon candidate

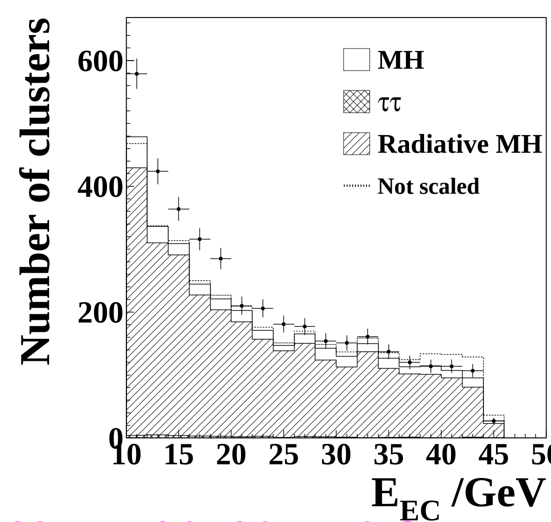
$\Longrightarrow$ 11.3 K hadronic events. Look at the hadronic subsystem

$$\sqrt{s'} = \sqrt{s}\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

Divide sample in 7 parts according to $\sqrt{s'}$. Largest background due to neutral hadrons. Use likelihood function using shower shape, isolation, $\cdots$



After isolation cuts

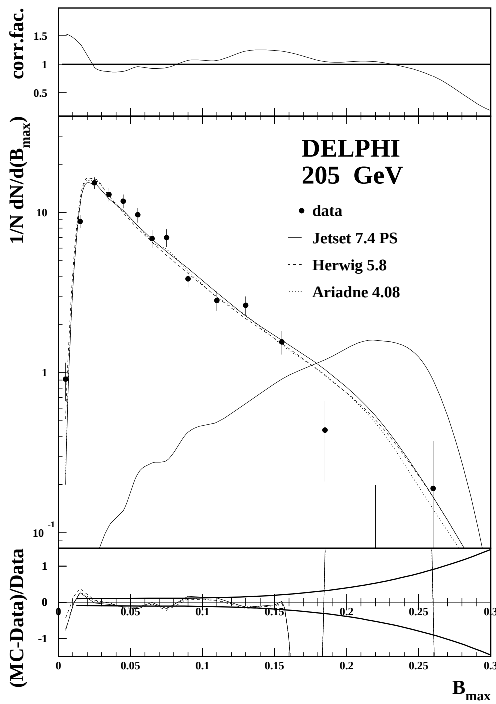


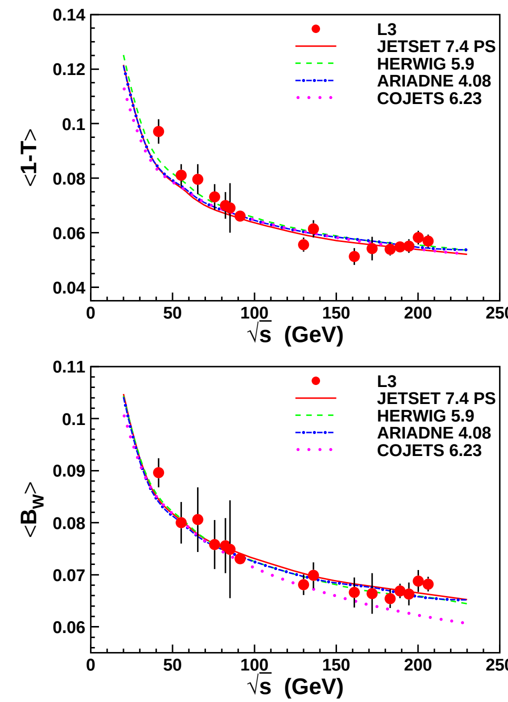
Using likelihood function

3.8 K events in 7 energy bins with BG between 0.6% and 6.6%

Event Shape Distributions

- Measure the distributions from charged & neutral particles
- Subtract known backgrounds (2-photon, 4-fermion, ISR)
- Correct for detector resolution and acceptance





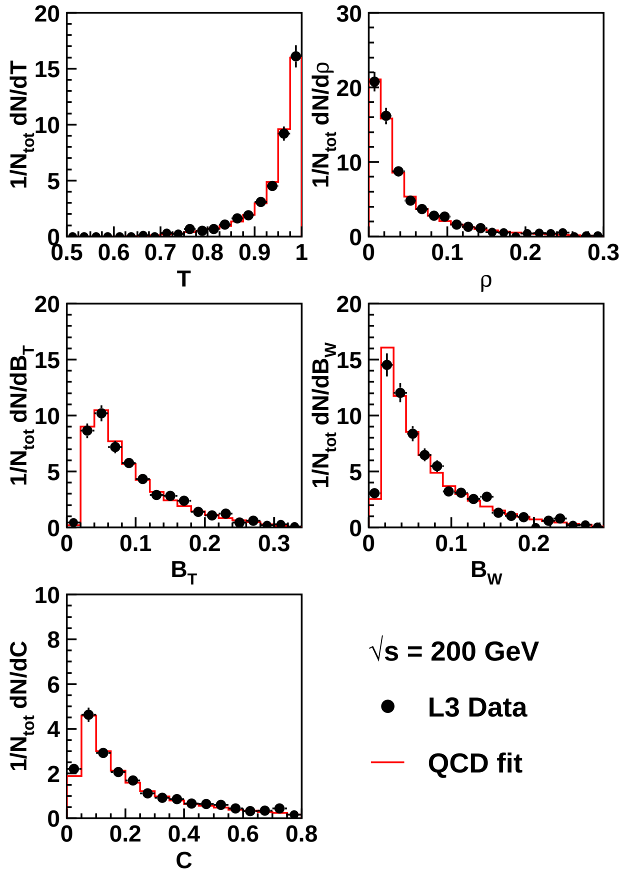
Small corrections ⇒ small systematic uncertainties

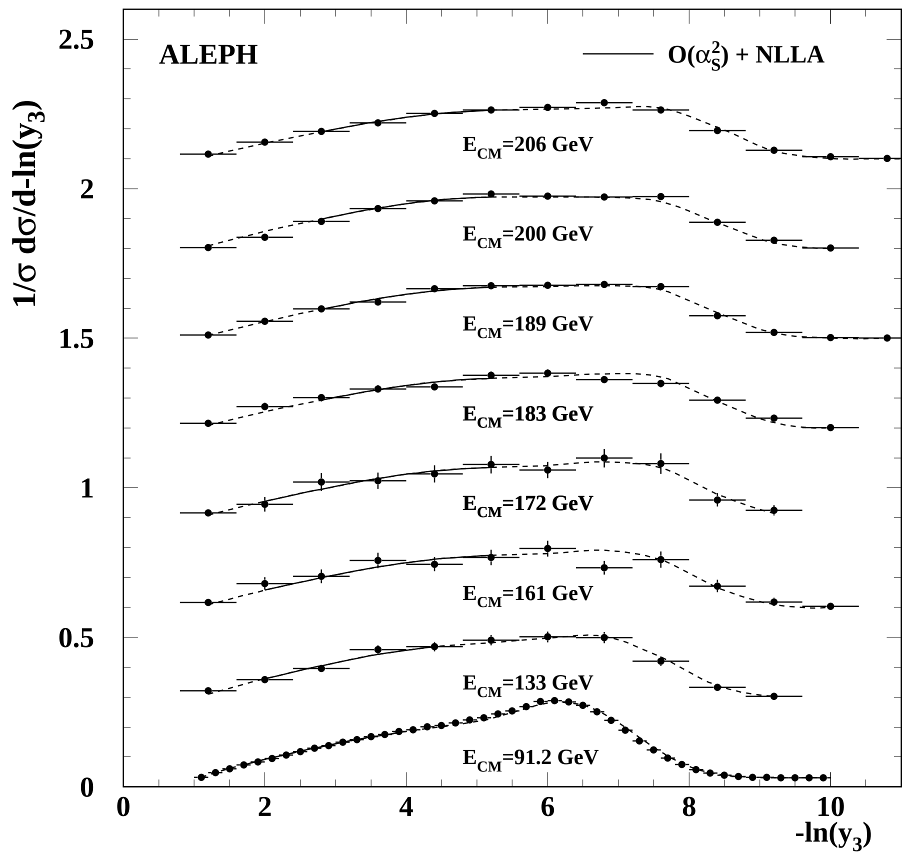
Energy evolution explained by Parton Shower models

Determination of α_s

Several groups have re-analysed their data in view of:

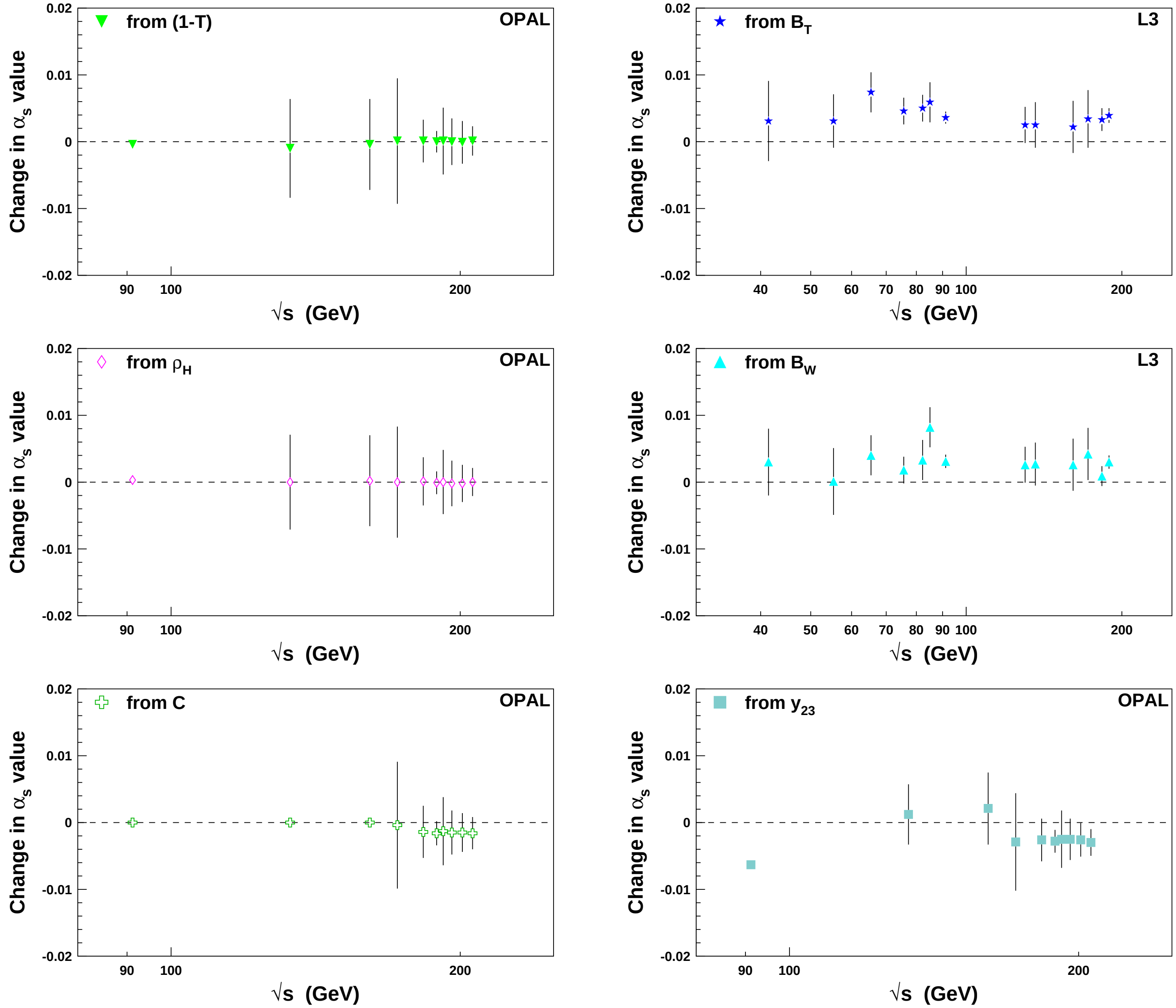
- Use of EVENT2 program (rather than EVENT) in determining the fixed order terms (better statistical precision)
- Use of more recent theoretical calculations for jet broadening variables (correct treatment of multiple gluon emission contribution) and for y_3 (completing the missing log terms due to multiple emission)
- Use of a more complete set of variables at all energies
- Use of Modified Log R matching scheme in determining the central value of α_s
- Use JETSET, HERWIG and ARIADNE in estimating the hadronisation correction





- pQCD predictions corrected for hadronisation effects fit data well
- use $p = 1$, $x_L = 1$ and renormalisation scale $x_\mu = 1$ for the central value of α_s

Effect of refits:



Classify errors on α_s as:

Statistical:

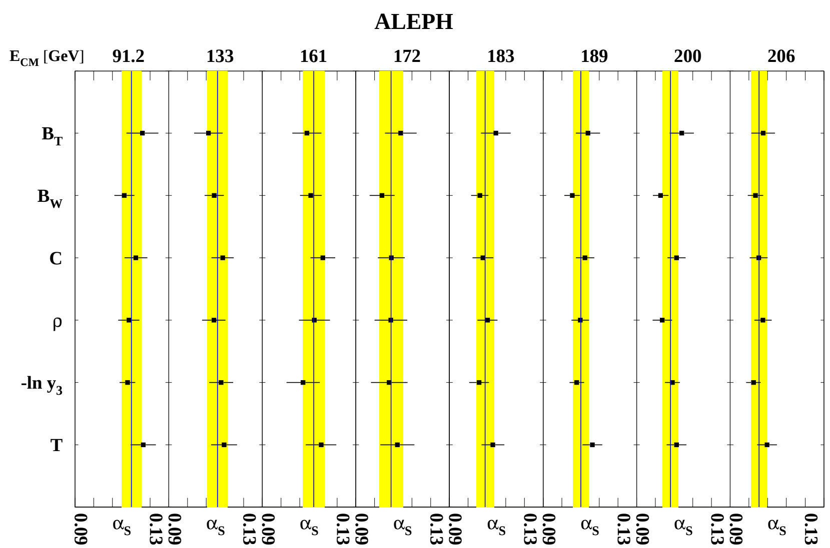
Experimental Systematic: estimated by varying (a) selection cuts, (b) methods for background estimates, (c) detector corrections, $\cdots$

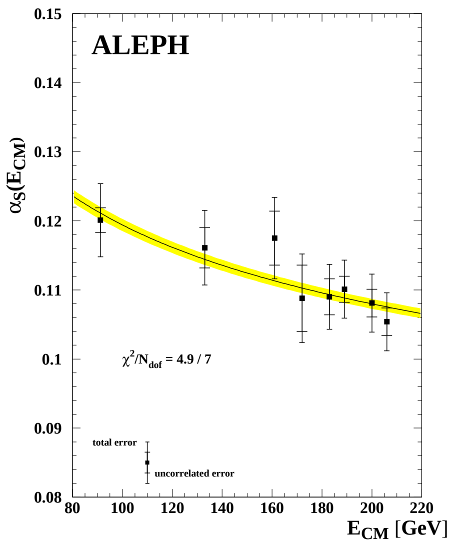
Hadronisation: estimated by changing the parton shower models PYTHIA/HERWIG/ARIADNE

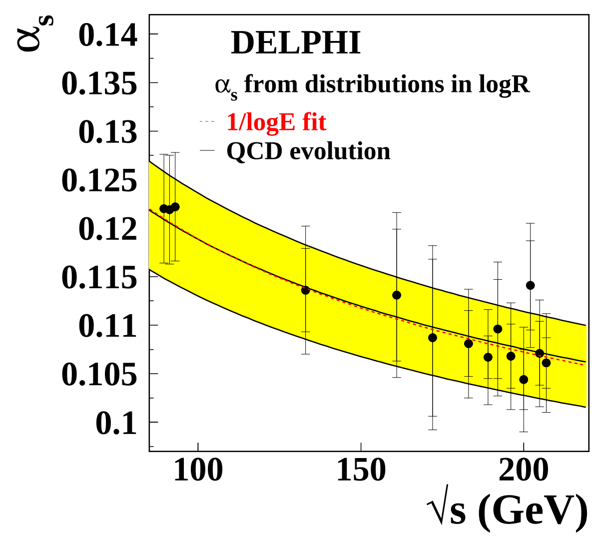
Theoretical: estimated by varying renormalisation scale, matching scheme, kinematic constraint, $\cdots$

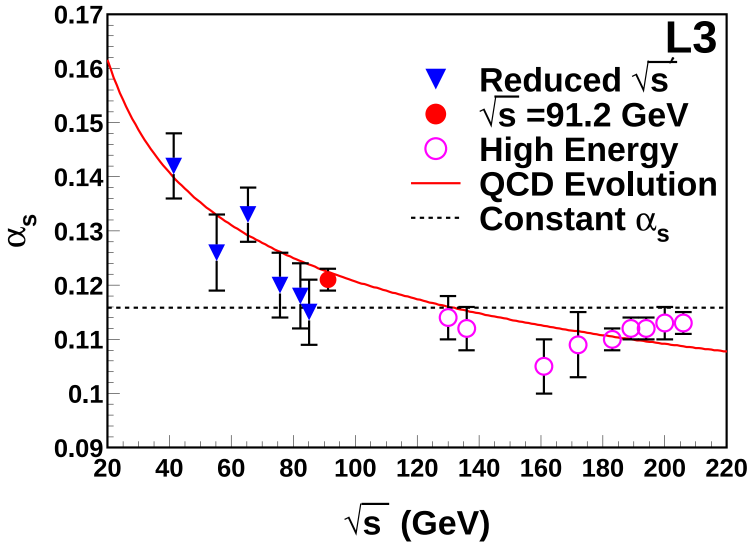
α_s determined from several event shape variables

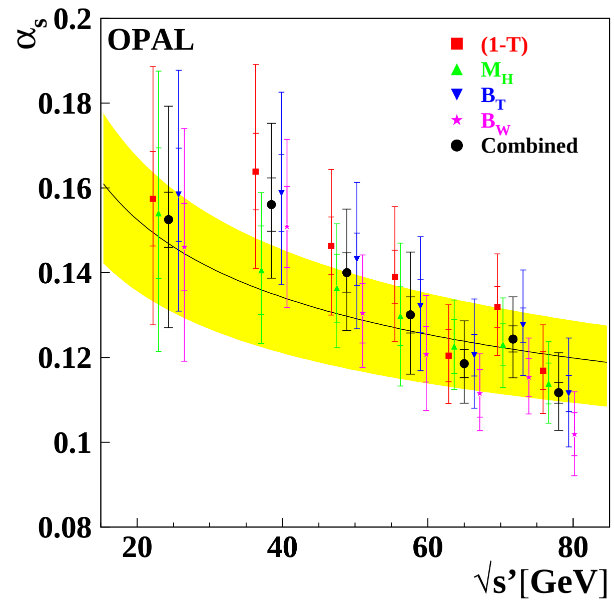
Combined to a single measurement by weighted/unweighted averages




ALEPH
$\alpha_S(E_{CM})$
χ^2/N_{dof} = 4.9 / 7
total error
uncorrelated error
E_{CM} [GeV]


DELPHI
α_s from distributions in logR
1/logE fit
QCD evolution
α_s
$\sqrt{s}$ (GeV)


L3
Reduced $\sqrt{s}$
$\sqrt{s}$ =91.2 GeV
High Energy
QCD Evolution
Constant α_s
α_s
$\sqrt{s}$ (GeV)


OPAL
(1-T)
M_H
B_T
B_W
Combined
α_s
$\sqrt{s'}$[GeV]

Summary

- The 4 LEP experiments have reanalysed their data on event shape variables for $e^+e^- \longrightarrow$ hadrons

- The 4 collaborations have reported their α_s values:

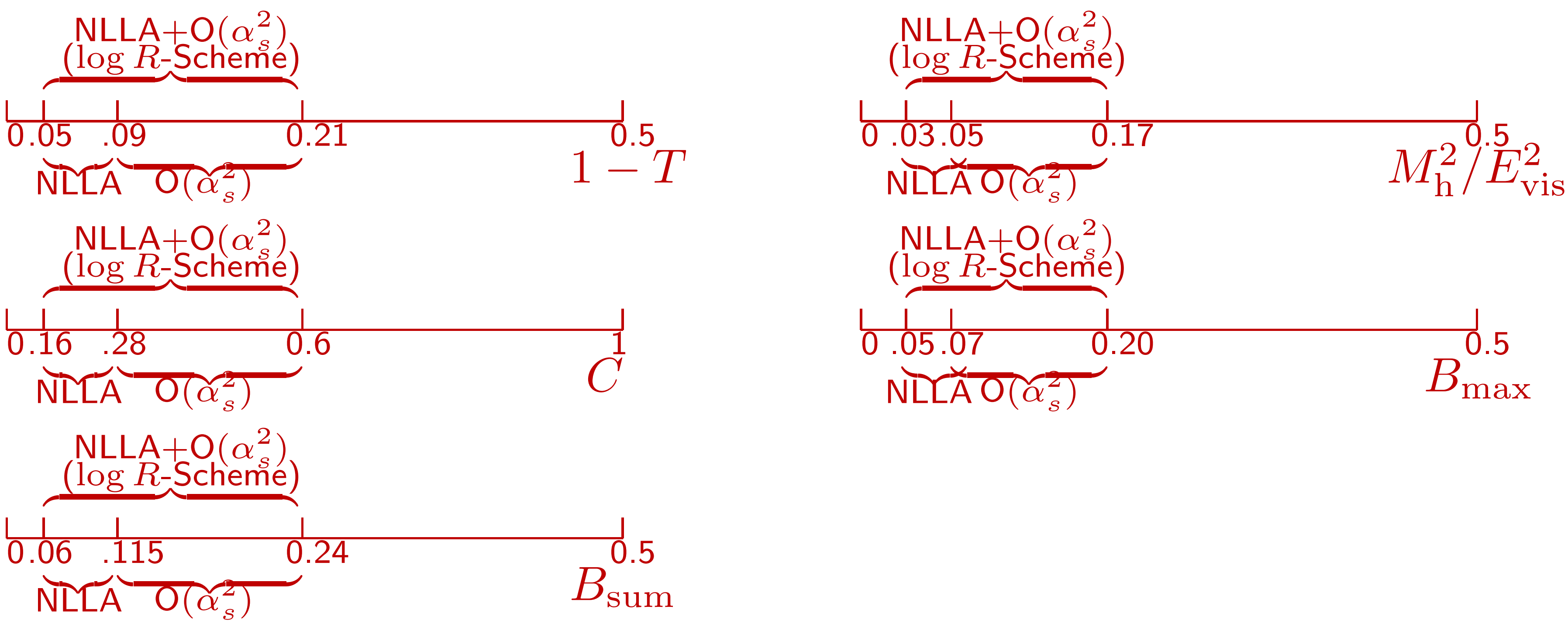
Experiment	$\alpha_s(M_Z)$
ALEPH	0.1214±0.0014±0.0046
DELPHI	0.1205±0.0021±0.0050
L3	0.1227±0.0012±0.0058
OPAL (Rad.)	$0.1176 \pm 0.0012\ ^{+0.0093}_{-0.0085}$

- A consistent treatment of these measurements and a better understanding of the theoretical uncertainties have been worked out by the LEP QCD Working Group

Fit ranges:

- the resummation calculation should be reliable
- the hadronization corrections would be small
- the signal-to-background ratio is favourable and detector correction small and rather uniform

DELPHI uses:



Papers summarised in this talk:

A172	ALEPH	Studies of QCD at Centre-of-Mass Energies between 91 and 209 GeV
A187	L3	Determination of α_s from Hadronic Event Shapes in e^+e^- Annihilation at $192 < \sqrt{s} < 209$ GeV
A303	DELPHI	The measurement of α_s from event shapes with the DELPHI detector at the highest LEP energies
A782	OPAL	Updated measurements of α_s using event shape observables
A785	OPAL	Measurement of α_s in Radiative Hadronic Events