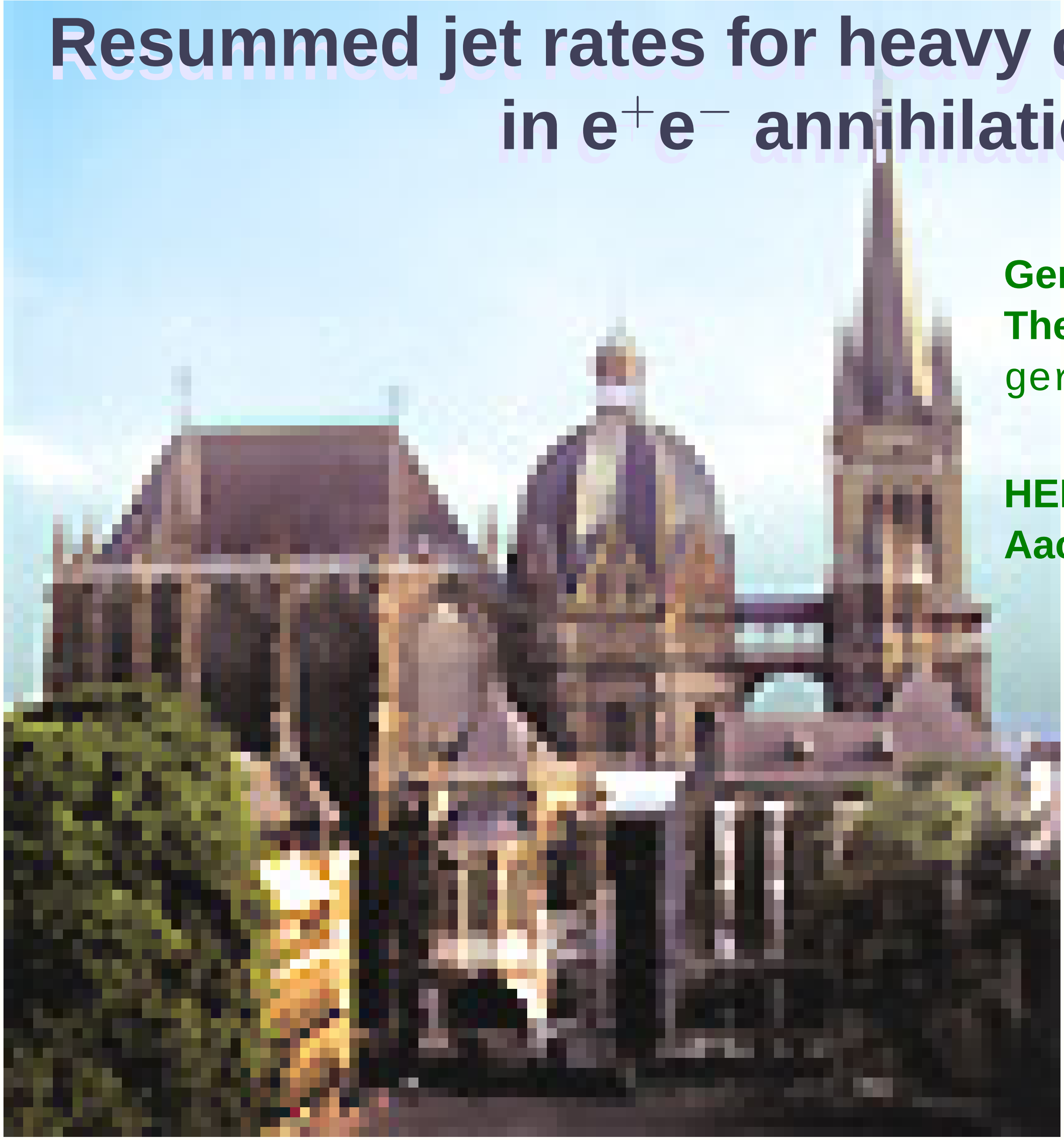
Resummed jet rates for heavy quark production in e^+e^- annihilation*



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* **F. Krauss, GR, hep-ph/0303038**

Outline

- Motivation
- Splitting Functions and Sudakov Form Factors with heavy particles
- Jet rates @ NLL in $k_\perp$ algorithms
- Numerical results
- Conclusions and outlook

m_b measurements @ high energy

A typical strategy to determine the mass of the bottom-quark at high energies (LEP, SLD) is to compare ratios of heavy to light observables:

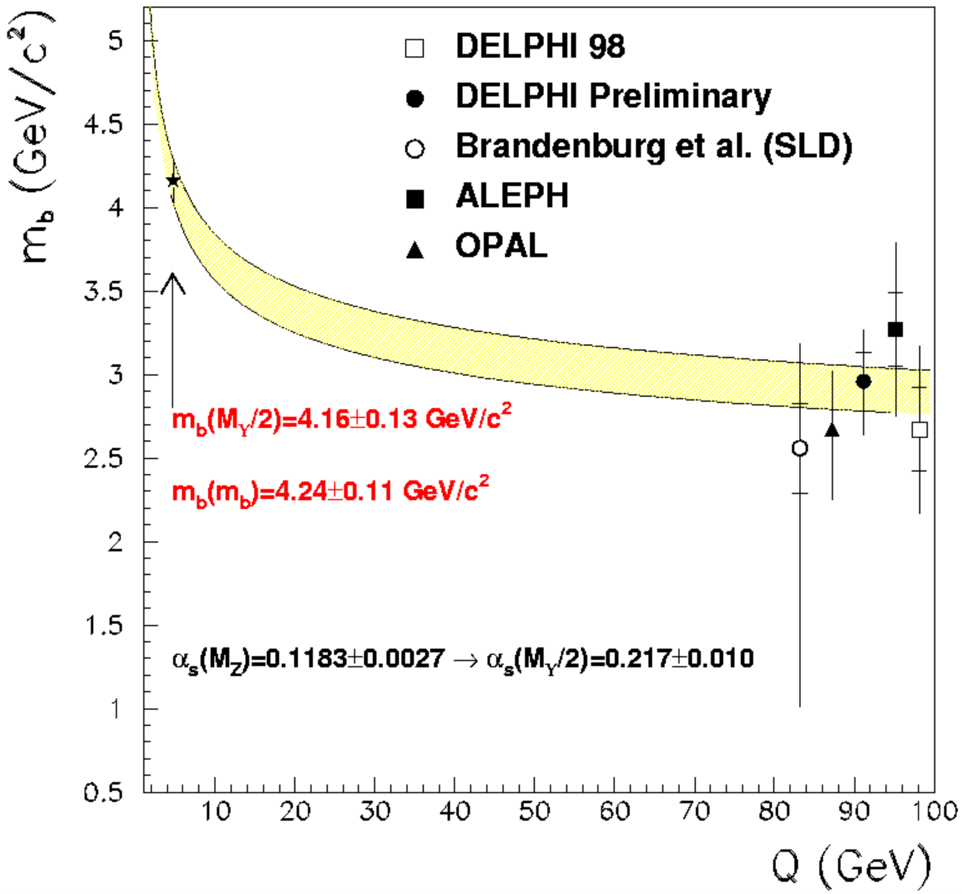
$$\frac{m_b^2}{M_Z^2} < .003 \; ,$$

but

$$\frac{1}{y_{\rm cut}} \times \frac{m_b^2}{M_Z^2} \sim \text{few} \, \% \; .$$

- ❖ n-jet cross-sections (different clustering algorithms)
- ❖ event-shape distrib. (moments)

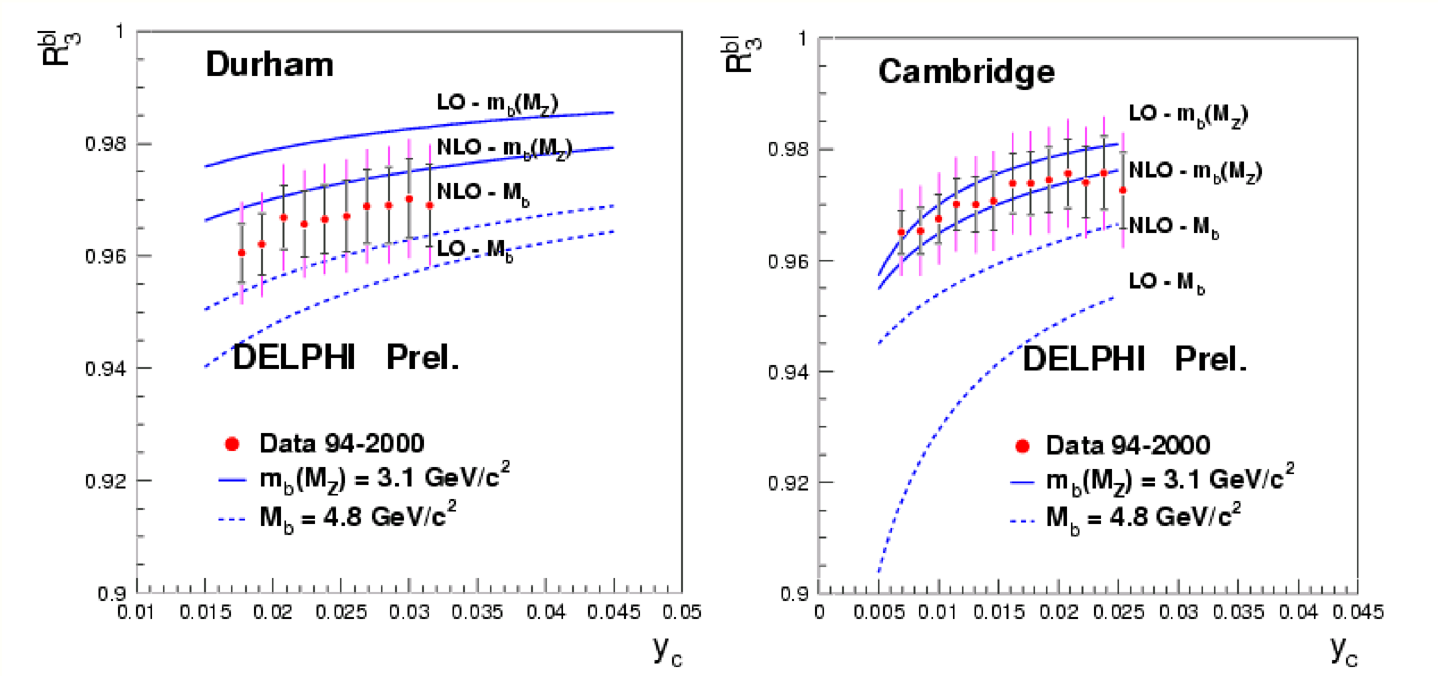
[M.J.Costa, see J.Fuster's talk]



Comparison with NLO calculations crucial

[GR-Santamaria-Bilenky, Bernreuther-Brandenburg-Uwer, Nason-Oleari].

Durham and Cambridge algorithms ($k_\perp$)



In both algorithms

$$y_{ij} = \frac{2 \min\left\{E_i^2,\, E_j^2\right\} \, (1 - \cos\theta_{ij})}{s} \lessgtr y_{\rm cut} \; .$$

- **minimize the effect of hadronization corrections**, and
- **exponentiation of leading (LL) and next-to-leading logarithms (NLL)** stemming from soft and collinear emission of secondary partons.

Motivation and Goals

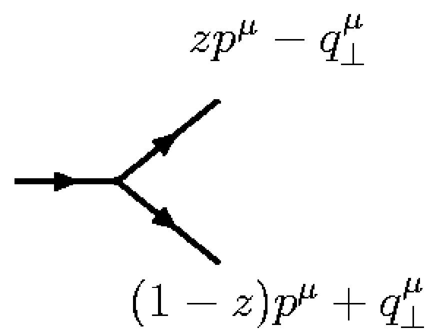
- ✔ Extend the validity of perturbative predictions to smaller $y_{\rm cut}$.
- ✔ Reduce renormalization scale dependence.
- ✔ A better understanding of the mass entering in the Monte Carlo.
- ✔ Predictions for top production at Linear Collider

For gluons emitted by the quark with $k_\perp < m$
➡ mass effects enhanced by **large logarithms** $\ln(m/k_\perp)$

❶ **Resummation:** Jet rates can be expressed, up to NLL accuracy, via **integrated splitting functions** and the corresponding **Sudakov form factors**.

❷ **Matching:** with fixed order calculations.

Splitting Functions with heavy particles

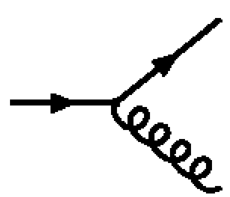


For the splitting process $Q \to Q(p_1) + g(p_2)$, Q being a heavy quark, $p_1^2 = m^2$, the matrix element factorizes as

$$|M(p_1, p_2; \cdots)|^2 \simeq |M(p_1 + p_2; \cdots)|^2 \, \frac{4\pi\alpha_s}{p_1 \cdot p_2} \, P_{QQ}(z, q)$$

$$\simeq |M(p_1 + p_2; \cdots)|^2 \, 8\pi\alpha_s \, \frac{z(1-z)}{\mathbf{q}^2 + (1-z)^2 m^2} \, P_{QQ}(z, q) \; ,$$

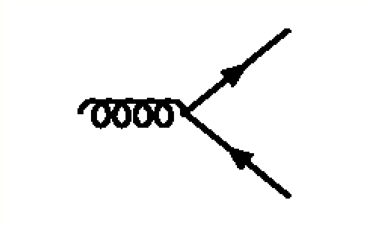
where the unregularized spin-averaged **splitting function** in $D = 4 - 2\epsilon$ dimensions (for our purposes $D = 4$) is given by [see also Catani, Dokshitzer, ...]



$$P_{QQ}(z, q) = C_F \left[\frac{1+z^2}{1-z} - \epsilon\,(1-z) - \frac{m^2}{p_1 \cdot p_2} \right]$$

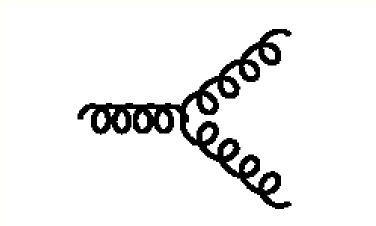
$$= C_F \left[\frac{1+z^2}{1-z} - \epsilon\,(1-z) - \frac{2z(1-z)m^2}{\mathbf{q}^2 + (1-z)^2 m^2} \right] \; .$$

Analogously, for the $g \to Q(p_1) + \bar{Q}(p_2)$ branching,



$$
\begin{aligned}
P_{gQ}(z,q) &= T_R \left[1 - \frac{2z(1-z)}{1-\epsilon} + \frac{2m^2}{(1-\epsilon)s_{12}} \right] \\
&= T_R \left[1 - \frac{2z(1-z)}{1-\epsilon} + \frac{2z(1-z)m^2}{(1-\epsilon)(\mathbf{q}^2+m^2)} \right] \; .
\end{aligned}
$$

For $g \to g(p_1) + g(p_2)$, no mass corrections at the lowest order



$$
P_{gg}(z) = C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] \; .
$$

Branching probabilities: massless quarks

Branching probabilities: probability to emit a parton with transverse momenta between q and $q+dq$.

For massless particles

$$
\begin{aligned}
\Gamma_Q(Q,q,m=0) &= \int_{q/Q}^{1-q/Q} dz\, P_{QQ}(z) = 2C_F\left(\ln\frac{Q}{q}-\frac{3}{4}\right)\,, \\
\Gamma_g(Q,q,m=0) &= \int_{q/Q}^{1-q/Q} dz\, P_{gg}(z) = 2C_A\left(\ln\frac{Q}{q}-\frac{11}{12}\right)\,, \\
\Gamma_f(Q,q,m=0) &= \int_{q/Q}^{1-q/Q} dz\, P_{gf}(z) = \frac{2n_f T_R}{3}\,,
\end{aligned}
$$

transverse momenta q and energy fraction z independent

Branching probabilities: heavy quarks

Propagator-like structures

$$\frac{1}{q^2} \quad \to \quad \frac{1}{q^2 + (1-z)^2 m^2}, \quad \frac{1}{q^2 + m^2}\,.$$

The **branching probabilities** for heavy quarks can then be defined through

$$\begin{aligned}
\Gamma_Q(Q,q,m) &= \int\limits_{q/Q}^{1-q/Q} dz \frac{q^2}{q^2 + (1-z)^2 m^2} P_{QQ}(z,q) \\
&= \Gamma_Q(Q,q,m=0) \\
&+ C_F \left[\frac{1}{2} - \frac{q}{m} \arctan\left(\frac{m}{q}\right) - \frac{2m^2 - q^2}{2m^2} \ln\left(\frac{m^2+q^2}{q^2}\right)\right], \\
\Gamma_f(Q,q,m) &= \int\limits_{q/Q}^{1-q/Q} dz \frac{q^2}{q^2 + m^2} P_{gQ}(z,q) = T_R \frac{q^2}{q^2+m^2}\left[1 - \frac{1}{3}\frac{q^2}{q^2+m^2}\right].
\end{aligned}$$

Branching probabilities: heavy quarks

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\end{aligned}$$

Correct limits:
$q < m : \quad \ln(m^2/q^2)$
$q > m : \quad m^2/q^2$

Sudakov form factors

Sudakov form factors: probability for a parton experiencing no emission of a secondary parton between transverse momentum scales Q down to Q_0

$$\Delta_Q(Q,Q_0) = \exp\left[-\int_{Q_0}^{Q}\frac{dq}{q}\frac{\alpha_s(q)}{\pi}\Gamma_Q(Q,q)\right] ,$$

$$\Delta_g(Q,Q_0) = \exp\left[-\int_{Q_0}^{Q}\frac{dq}{q}\frac{\alpha_s(q)}{\pi}\left(\Gamma_g(Q,q)+\Gamma_f(Q,q)\right)\right] ,$$

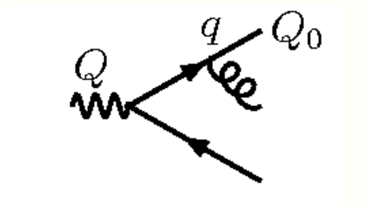
$$\Delta_f(Q,Q_0) = \left[\Delta_Q(Q,Q_0)\right]^2/\Delta_g(Q,Q_0) \, .$$

Jet rates @ NLL

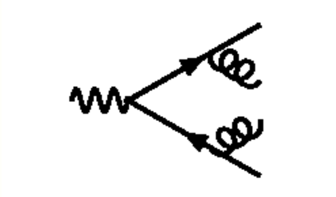
Rates for n-jet events in $k_\perp$ algorithms at NLL accuracy via Sudakov form factors and branching probabilities [Catani, Webber, Dokshitzer, Fiorani]

$$\mathcal{R}_2 = \left[\Delta_Q(Q,Q_0)\right]^2 \; ,$$

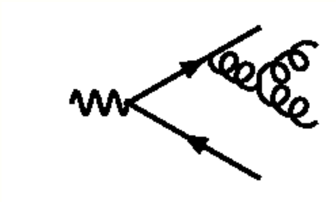
$$\mathcal{R}_3 = 2\left[\Delta_Q(Q,Q_0)\right]^2 \int_{Q_0}^{Q} \frac{dq}{q}\frac{\alpha_s(q)}{\pi}\Gamma_Q(Q,q)\Delta_g(q,Q_0)\; ,$$



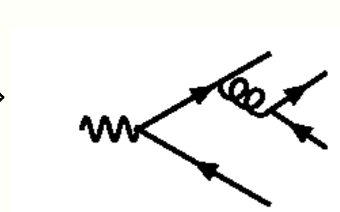
$$\mathcal{R}_4 = 2\left[\Delta_Q(Q,Q_0)\right]^2 \left\{ \left[\int_{Q_0}^{Q} \frac{dq}{q}\frac{\alpha_s(q)}{\pi}\Gamma_Q(Q,q)\Delta_g(q,Q_0)\right]^2 \right.$$



$$+ \int_{Q_0}^{Q} \frac{dq}{q}\left[\frac{\alpha_s(q)}{\pi}\Gamma_Q(Q,q)\Delta_g(q,Q_0)\int_{Q_0}^{q}\frac{dq'}{q'}\frac{\alpha_s(q')}{\pi}\Gamma_g(q,q')\Delta_g(q',Q_0)\right]$$



$$\left. + \int_{Q_0}^{Q} \frac{dq}{q}\left[\frac{\alpha_s(q)}{\pi}\Gamma_Q(Q,q)\Delta_g(q,Q_0)\int_{Q_0}^{q}\frac{dq'}{q'}\frac{\alpha_s(q')}{\pi}\Gamma_f(q,q')\Delta_f(q',Q_0)\right] \right\}$$



where $Q_0^2 = y_{\rm cut} Q^2$, with Q the c.m. energy of the colliding e^+e^-.

NLL logarithms @ fixed order

For $Q \gg m \gg Q_0$, and expanding in α_s: $L_y = \ln(1/y_{\text{cut}})$, $L_m = \ln(m^2/Q_0^2)$,

$$\begin{aligned}
\mathcal{R}_2 &= 1 + \left(\frac{\alpha_s(Q)}{\pi}\right)\left[-\frac{1}{2}C_F(L_y^2 - L_m^2) + \frac{3}{2}C_F L_y + \frac{1}{2}C_F L_m\right] \\
&+ \left(\frac{\alpha_s(Q)}{\pi}\right)^2\left[\frac{1}{8}C_F^2\left(L_y^2 - L_m^2\right)^2 - C_F^2\left(L_y^2 - L_m^2\right)\left(\frac{3}{4}L_y - \frac{1}{4}L_m\right)\right. \\
&\left.-\frac{1}{3}\beta_n C_F\left(L_y^3 - \frac{3}{2}L_y L_m^2 + \frac{1}{2}L_m^3\right) - \frac{1}{3}(\beta_n - \beta_{n-1})C_F L_m^3\right] , \\
\mathcal{R}_3 &= \left(\frac{\alpha_s(Q)}{\pi}\right)\left[\frac{1}{2}C_F(L_y^2 - L_m^2) - \frac{3}{2}C_F L_y - \frac{1}{2}C_F L_m\right] \\
&+ \left(\frac{\alpha_s(Q)}{\pi}\right)^2\left[-\frac{1}{4}C_F^2\left(L_y^2 - L_m^2\right)^2 - \frac{1}{48}C_F C_A\left(L_y^4 - L_m^4\right)\right. \\
&+\frac{1}{2}C_F^2\left(L_y^2 - L_m^2\right)(3L_y - L_m) + \frac{1}{24}C_F C_A\left(3L_y^3 - L_m^3\right) \\
&\left.+\frac{1}{2}\beta_n C_F L_y\left(L_y^2 - L_m^2\right) + \frac{1}{6}(\beta_n - \beta_{n-1})C_F L_m\left(L_y^2 - L_y L_m + 2L_m^2\right)\right] , \\
\mathcal{R}_4 &= 1 - \mathcal{R}_2 - \mathcal{R}_3 ,
\end{aligned}$$

NLL logarithms @ fixed order

For $Q \gg m \gg Q_0$, and expanding in α_s: $L_y = \ln(1/y_{\rm cut})$, $L_m = \ln(m^2/Q_0^2)$,

$$\mathcal{R}_2 = 1 + \left(\frac{\alpha_s(Q)}{\pi}\right) \left[-\frac{1}{2}C_F(L_y^2 - L_m^2) + \frac{3}{2}C_F L_y + \frac{1}{2}C_F L_m\right]$$

$$+ \left(\frac{\alpha_s(Q)}{\pi}\right)^2 \left[\frac{1}{8}C_F^2\left(L_y^2 - L_m^2\right)^2 - C_F^2\left(L_y^2 - L_m^2\right)\left(\frac{3}{4}L_y - \frac{1}{4}L_m\right)\right.$$

$$\left. -\frac{1}{3}\beta_n C_F\left(L_y^3 - \frac{3}{2}L_y L_m^2 + \frac{1}{2}L_m^3\right) - \frac{1}{3}(\beta_n - \beta_{n-1})C_F L_m^3\right] ,$$

$$\mathcal{R}_3 = \left(\frac{\alpha_s(Q)}{\pi}\right)\left[\frac{1}{2}C_F(L_y^2 - L_m^2) - \frac{3}{2}C_F L_y - \frac{1}{2}C_F L_m\right]$$

$$+ \left(\frac{\alpha_s(Q)}{\pi}\right)^2 \left[-\frac{1}{4}C_F^2\left(L_y^2 - L_m^2\right)^2 - \frac{1}{48}C_F C_A\left(L_y^4 - L_m^4\right)\right.$$

$$+\frac{1}{2}C_F^2\left(L_y^2 - L_m^2\right)(3L_y - L_m) + \frac{1}{24}C_F C_A \ldots$$

$$+\frac{1}{2}\beta_n C_F L_y\left(L_y^2 - L_m^2\right) + \frac{1}{6}(\beta_n - \beta_{n-1})C \ldots$$

$$\mathcal{R}_4 = 1 - \mathcal{R}_2 - \mathcal{R}_3 ,$$

Partial cancellation of LL

massless result [Catani *et al.*]
for $L_m \to 0$

Dead cone approximation

Dead cone [Dokshitzer *et al.*] at leading logarithmic order (LL), there is no radiation of soft and collinear gluons off heavy quarks.

$$P_{QQ}(z,q) = C_F \left[\frac{1+z^2}{1-z} - \epsilon\,(1-z) - \frac{2z(1-z)m^2}{\mathbf{q}^2 + (1-z)^2 m^2} \right] \,,$$

has the correct massless limit and matches the **LL splitting function**

$$P_{QQ}^{LL}(z) = C_F \left[\frac{2}{1-z} \right] \,,$$

but only if $\mathbf{q}^2 > (1-z)^2 m^2$
➥ $z > 1 - q/m$.

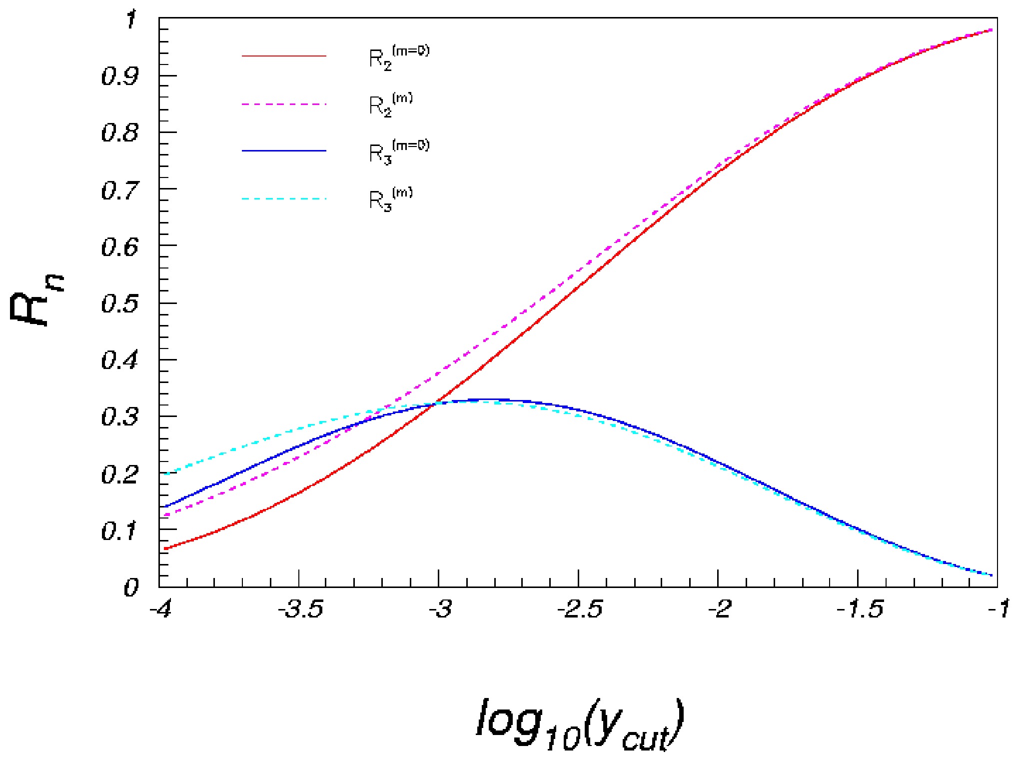
This can be expressed via the modified integrated splitting function

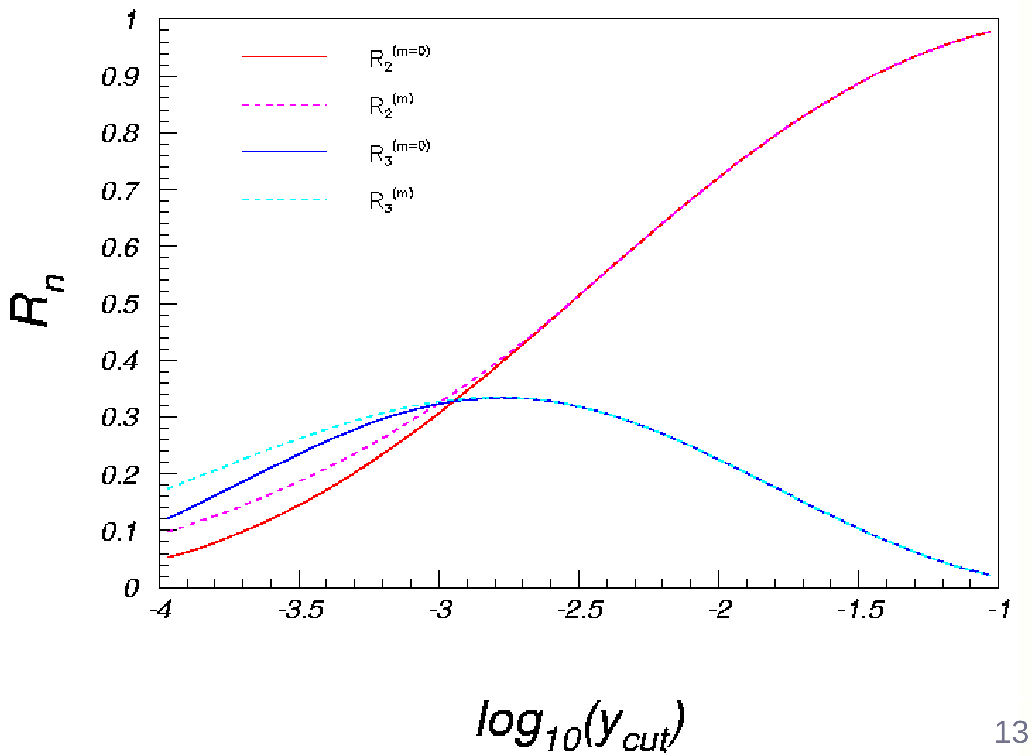
$$\begin{aligned} \Gamma_Q^{\mathrm{d.c.}}(Q,q,m) &= \int_{1-q/m}^{1-q/Q} dz\, P_{QQ}(z,q,m=0) \\ &= \Gamma_Q(Q,q,m=0) + 2C_F \ln\left(\frac{q}{m}\right) \Theta(m-q) \,. \end{aligned}$$

Bottom production @ LEP1 (91 GeV)

two and three-jet rates (m_b = 5 GeV)

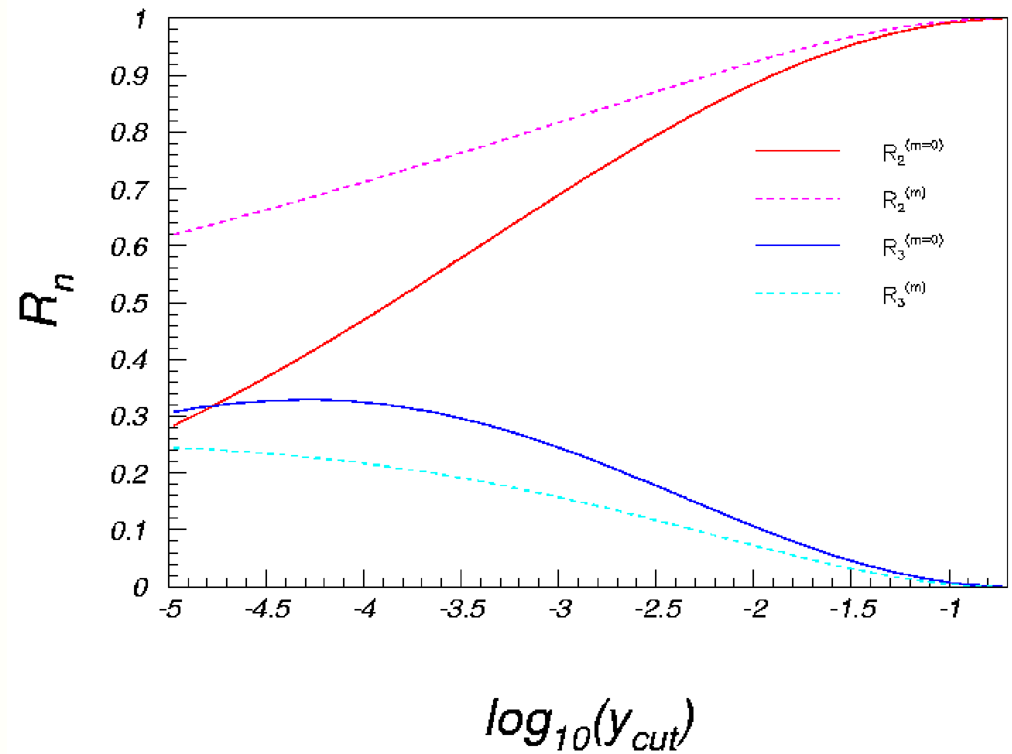
- Correct behaviour: heavy quarks radiate less gluons than light quarks ($k_\perp > m_b$)
- **'dead cone'** approximation: only $k_\perp < m_b$



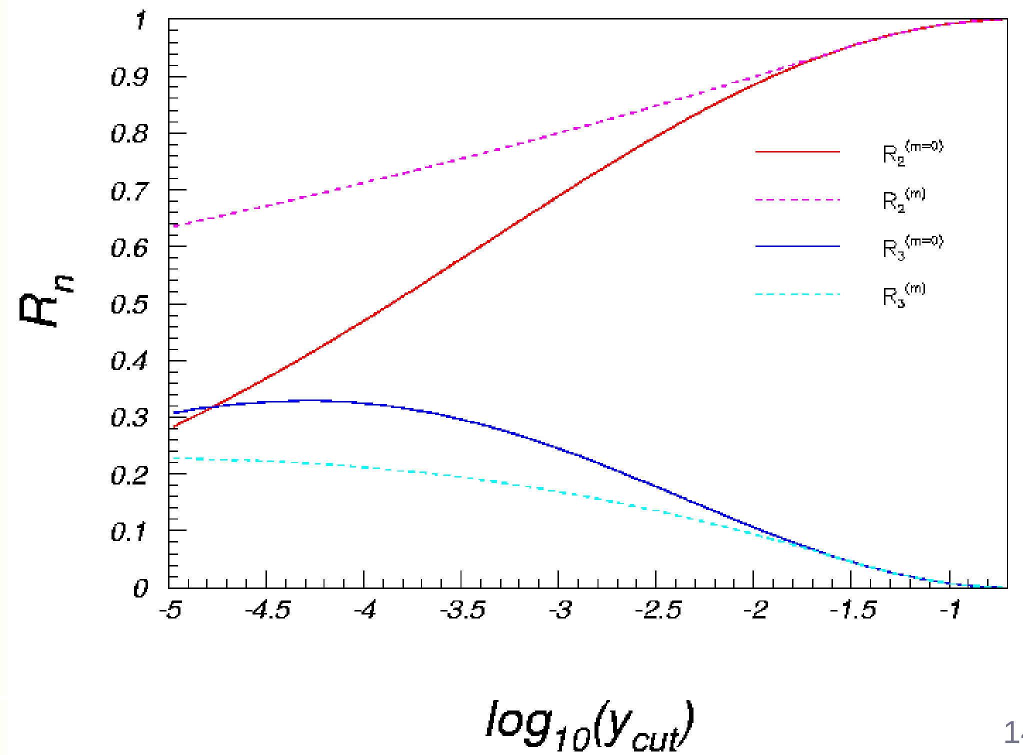


Top production @ Linear Collider (1 TeV)

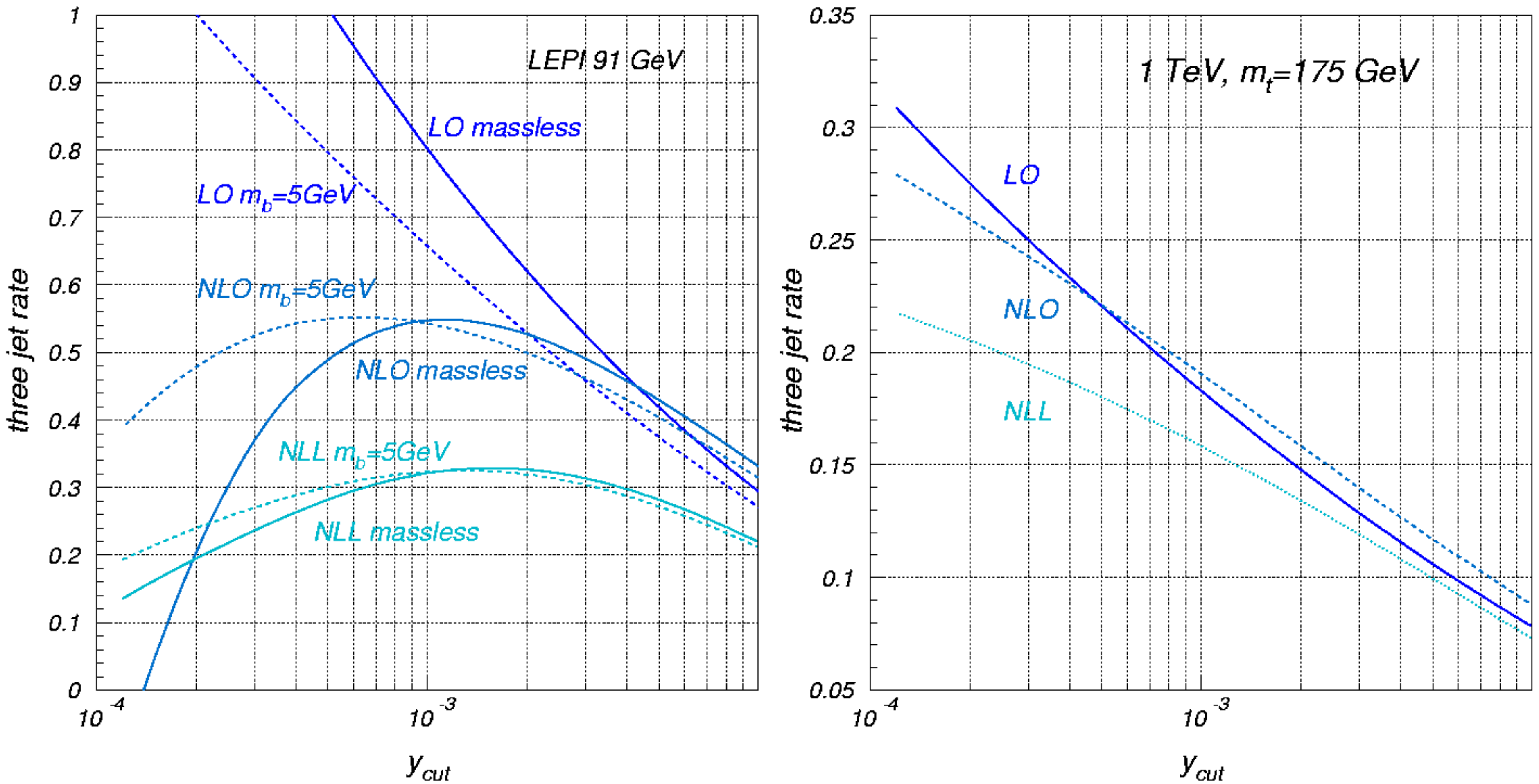
two and three-jet rates (m_t = 175 GeV)



- Huge corrections.
- Strong cancelation of LL, subleading effects important.



Fixed order vs resummed



✔ Matching to be done but already good description of the shape

Conclusions and outlook

- Sudakov form factors involving heavy quarks have been employed to predict jet rates in e^+e^- annihilation into hadrons.

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- Sudakov form factors involving heavy quarks have been employed to predict jet rates in e^+e^- annihilation into hadrons.
- Exhibit the correct logarithmic behaviour and thus provide a good estimate for the size of mass effects.
- Matching between fixed-order calculations and resummed results in progress.