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# Coupling $g_{f_0 K^+ K^-}$ and the structure of $f_0(980)$

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- Scalar mesons: what is their structure?
- Some hints to discriminate among possible scenarios
- The strong coupling $g_{f_0 K^+ K^-}$
- Conclusions and perspectives

**Based on a work in collaboration with P. Colangelo**

Quark Model: Hadrons fit in multiplets

Mesons= $q\bar{q}$ states

## What about scalar mesons?

- There are more scalars than can fit into one quark model multiplet
- They have short lifetimes and large couplings to hadronic states
- They often seem to contradict the OZI rule

Possibilities:

- $q\bar{q}$ states
- glueballs
- quark-gluon admixtures
- multiquark states
- composite states of hadrons $\left(\left|K\bar{K}\right\rangle, \left|\pi\eta\right\rangle, \ldots\right)$

## The case of $f_0(980)$ and $a_0(980)$

★ Quark model interpretation: $f_0 = s\bar{s}, \quad a_0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$

This does not explain the mass degeneracy

$$\begin{cases} M_{f_0} = 980 \pm 10 \quad MeV \\ M_{a_0} = 984.7 \pm 1.2 \quad MeV \end{cases}$$

★ Four quark state: $q\bar{q}q\bar{q}$

- Nucleon-like, i.e. bound state of quarks:

$$f_0 = s\bar{s}\frac{(u\bar{u} + d\bar{d})}{\sqrt{2}} \qquad a_0 = s\bar{s}\frac{(u\bar{u} - d\bar{d})}{\sqrt{2}}$$

⟹ Point-like objects

- Deuteron-like, i.e. bound state of hadrons:

$$f_0 = \left|K\bar{K}\right\rangle \qquad a_0 = \left|\pi\eta\right\rangle$$

⟹ Extended objects

The radiative decay $\phi \to f_0\gamma$ may help to discriminate among the various possibilities

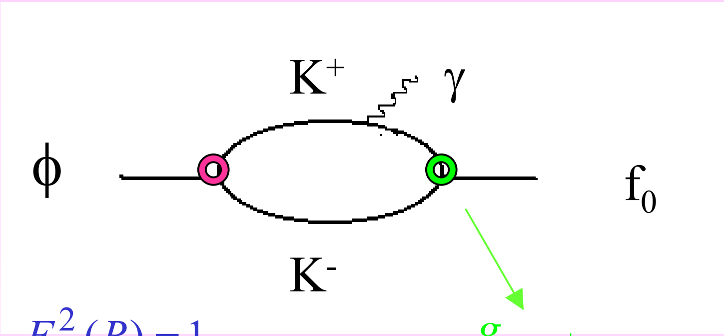
- If $f_0 = s\bar{s}$ the dominant mechanism is the direct transition and it is expected

$$BR(\phi \to f_0\gamma) \approx O(10^{-5})$$

(The corresponding decay to $a_0$ would be OZI suppressed in this case)

- In the 4q and $K\bar{K}$ scenarios it is assumed that the decay proceeds through a kaon loop and:

$$BR(\phi \to f_0\gamma) = (2.5 \pm 0.5) \times 10^{-4} \cdot F^2(R)$$




$g_{f_0K^+K^-}$

where for a point-like object $(R < \Lambda_{QCD}^{-1})$: $F^2(R) = 1$

while for a $K\bar{K}$ molecule $(R > \Lambda_{QCD}^{-1})$: $F^2(R) < 1$

- $g_{\phi K^+K^-}$ (known from exp data)
- $g_{f_0K^+K^-}$

Close, Isgur and Kumano
Achasov and Ivanchenko
Close and Kirk

In the composite state scenario, one could suppose that the pure $q\bar{q}$ state is enriched by strong interaction with other components (such as $|K\bar{K}\rangle, |\pi\eta\rangle$ )
(*Hadronic Dressing*)

M.Boglione, M.R.Pennington
PRL 79 (97)1998

This would also explain why scalar mesons seem to contradict the OZI rule:
the two hadrons composing the state in which they spend much of their lifetime may annihilate to $q\bar{q}$ leading to a subsequent OZI allowed decay

For example, the decay $\phi \to a_0\gamma$ would be Zweig suppressed if $a_0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$
but it could nevertheless proceed through kaon loop
(Zweig violating process)

Another interesting quantity is: $R_\phi = \dfrac{BR(\phi \to f_0\gamma)}{BR(\phi \to a_0\gamma)}$

- In the $q\bar{q}$ scenario, due to the OZI suppression of the decay to $a_0$, it could even be $R_\phi \approx 10$

- In the $K\bar{K}$ molecule scenario, one may write:

$$|f_0\rangle = \cos\vartheta |K^+K^-\rangle + \sin\vartheta |K^0\bar{K}^0\rangle$$

$$|a_0\rangle = \sin\vartheta |K^+K^-\rangle - \cos\vartheta |K^0\bar{K}^0\rangle$$

And $R_\phi$ depends on $\text{ctg}^2\vartheta$ and on $g_{f_0K^+K^-}$

If $f_0$ and $a_0$ are isospin eigenstates with I=0,1 ⟹ $\vartheta = \frac{\pi}{4}$

and $R_\phi \cong 1$

- In the 4q scenario this ratio depends on the details of the structure of the particles

Close, Isgur and Kumano

Several interesting quantities depend on $g_{f_0 K^+ K^-}$

## Calculation of the strong coupling $g_{f_0 K^+ K^-}$

Method: Light Cone QCD Sum Rules

Starting point: $T_\mu(p,q) = i\int d^4x \quad e^{ip\cdot x}\langle K^+(q)|T[J_\mu^K(x)J_{f_0}(0)]|0\rangle$

$$J_\mu^K = \bar{u}\gamma_\mu\gamma_5 s \qquad J_{f_0} = \bar{s}s$$

$T_\mu$ can be written in terms of two independent invariant functions:

$$T_\mu(p,q) = i\,T_1(p^2,(p+q)^2)p_\mu + T_2(p^2,(p+q)^2)q_\mu$$

The method consists in representing $T_\mu$ in terms of the contribution of hadrons (one particle states and the continuum) having non-vanishing matrix elements with the vacuum and the currents $J_\mu^K$, $J_{f_0}$ and matching such a representation with the QCD expression

The hadronic representation is:

$$T_1^{HAD}(p^2,(p+q)^2) = \frac{1}{(M_K^2 - p^2)[M_{f_0}^2 - (p+q)^2]} g_{f_0 K^+ K^-} M_{f_0} \tilde{f} \, f_K + \ldots$$

where:

$$\langle K^+(q) K^-(p) | f_0(p+q) \rangle = g_{f_0 K^+ K^-}$$

$$\langle f_0(980)(p+q) | J_{f_0} | 0 \rangle = M_{f_0} \tilde{f}$$

$$\langle 0 | J_\mu^K | K(p) \rangle = i f_K p_\mu$$

The QCD representation is obtained expanding the T-product near the light cone ($x^2$=0) in terms of non-local operators, the matrix elements of which are defined by wave functions associated to operators of increasing twist.

Example:

$$\langle K(q)|\overline{q}(x)i\gamma_5 s(0)|0\rangle = \frac{f_K M_K^2}{m_s}\int_0^1 du \quad e^{iuq\cdot x}\,\varphi_P(u)$$

To twist four accuracy:

$$T_1^{QCD}(p^2,(p+q)^2) = f_K\int_0^1 du\left\{\frac{M_K^2}{m_s}\varphi_P(u)\frac{1}{m_s^2-(p+uq)^2}\right.$$
$$-2\left[m_s g_2(u)+\frac{M_K^2}{6m_s}\varphi_\sigma(u)(p\cdot q+uM_K^2)\right]\left.\frac{1}{[m_s^2-(p+uq)^2]^2}\right\}$$
$$+f_{3K}M_K^2\int_0^1 du\left(2\text{v}+\frac{1}{2}\right)\int D\alpha_i\varphi_{3K}(\alpha_i)\frac{1}{\left\{[p+q(\alpha_1+v\alpha_3)]^2-m_s^2\right\}^2}$$
$$-4f_K m_s M_K^2\left\{\int_0^1 dv(v-1)\int d\alpha_3\hat{\psi}(\alpha_3)\frac{1}{\left\{m_s^2-[p+q((v-1)\alpha_3+1)]^2\right\}^3}\right.$$
$$\left.+\int_0^1 d\alpha_3\int_0^{1-\alpha_3} d\alpha_1\hat{\phi}(\alpha_i)\frac{1}{\left\{m_s^2-[p+q(\alpha_1+v\alpha_3)]^2\right\}^3}\right\}$$

# Kaon distribution amplitudes

$$< K(q)|\bar{u}(x) i\gamma_5 s(0)|0> \; = \; \frac{f_K M_K^2}{m_s} \int_0^1 du \; e^{iuq\cdot x} \varphi_p(u) \; ,$$

$$< K(q)|\bar{u}(x)\sigma_{\mu\nu}\gamma_5 s(0)|0> \; = \; i(q_\mu x_\nu - q_\nu x_\mu)\frac{f_K M_K^2}{6m_s} \int_0^1 du \; e^{iuq\cdot x} \varphi_\sigma(u)$$

$$\begin{aligned} < K(q)|\bar{u}(x)\gamma_\mu\gamma_5 s(0)|0 >= & - \; i f_K q_\mu \int_0^1 du \; e^{iuq\cdot x}[\varphi_K(u) + x^2 g_1(u)] \\ & + \; f_K(x_\mu - \frac{q_\mu x^2}{q\cdot x}) \int_0^1 du \; e^{iuq\cdot x} g_2(u) \; . \end{aligned}$$

$$< K(q)|\bar{u}(x)\sigma_{\alpha\beta}\gamma_5 g_s G_{\mu\nu}(vx)s(0)|0 >=$$
$$if_{3K}[(q_\mu q_\alpha g_{\nu\beta} - q_\nu q_\alpha g_{\mu\beta}) - (q_\mu q_\beta g_{\nu\alpha} - q_\nu q_\beta g_{\mu\alpha})] \int \mathcal{D}\alpha_i \; \varphi_{3K}(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}$$

$$< K(q)|\bar{u}(x)\gamma_\mu\gamma_5 g_s G_{\alpha\beta}(vx)s(0)|0 >=$$
$$f_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\right) - q_\alpha\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\right)\right] \int \mathcal{D}\alpha_i \varphi_\perp(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}$$
$$+ f_K \frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \varphi_\parallel(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}$$

$$< K(q)|\bar{u}(x)\gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx)s(0)|0 >=$$
$$if_K\left[q_\beta\left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{q\cdot x}\right) - q_\alpha\left(g_{\beta\mu} - \frac{x_\beta q_\mu}{q\cdot x}\right)\right] \int \mathcal{D}\alpha_i \tilde{\varphi}_\perp(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)}$$
$$+ if_K \frac{q_\mu}{q\cdot x}(q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha_i \tilde{\varphi}_\parallel(\alpha_i) e^{iq\cdot x(\alpha_1 + v\alpha_3)} \, .$$

## Some explicit expressions

$$\varphi_P(u,\mu)=\sum_k a_k^P(\mu)C_k^{\frac{1}{2}}(2u-1)$$

$$a_0^P=1 \quad a_2^P=30\eta_3-\frac{5}{2}\rho^2 \quad a_4^P=-3\eta_3\omega_3-\frac{27}{20}\rho^2-\frac{81}{10}\rho^2\tilde{a}_2$$

$$\tilde{a}_2=0.2 \qquad \eta_3=0.015 \qquad \omega_3=-3 \qquad (\mu=1 \quad GeV)$$

$$\varphi_\sigma(u,\mu)=6u(1-u)\sum_k a_k^\sigma(\mu)C_k^{\frac{3}{2}}(2u-1)$$

$$a_0^\sigma=1 \quad a_2^\sigma=5\eta_3-\frac{1}{2}\eta_3\omega_3-\frac{7}{20}\rho^2-\frac{3}{5}\rho^2\tilde{a}_2$$

$\rho^2=\frac{m_s^2}{M_K^2}$ ⟶ takes into account meson mass corrections

$C_k^{\frac{1}{2}},\ C_k^{\frac{3}{2}}$ ⟶ Gegenbauer polynomials

The sum rule follows from the approximate equality:

$$T_1^{HAD}(p^2,(p+q)^2) = T_1^{QCD}(p^2,(p+q)^2)$$

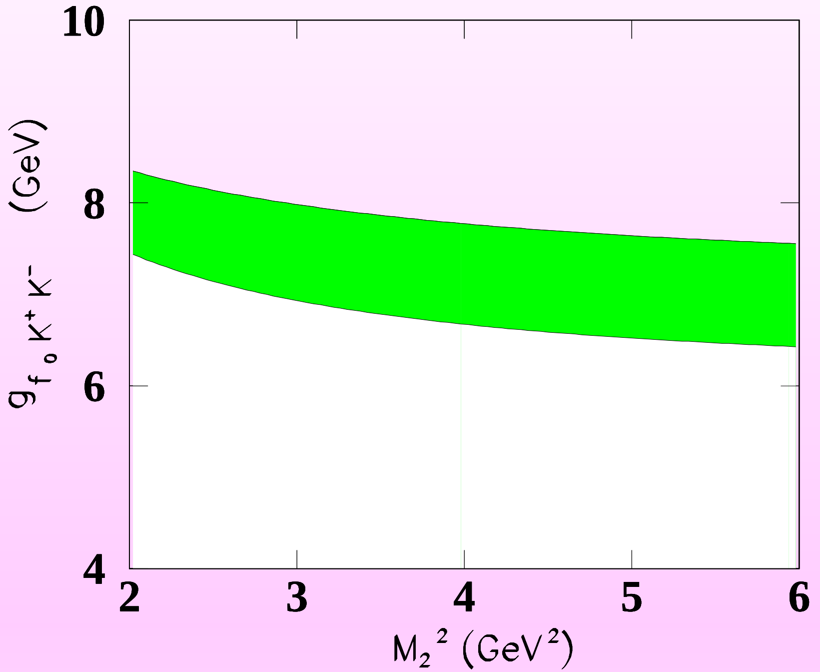
which can be improved applying a Borel transformation with respect to the variables $-p^2, -(p+q)^2$ :

$$B_{M_1^2} B_{M_2^2} \frac{(\lambda-1)!}{[m_s^2-(p+uq)^2]^{\lambda}} = \frac{(M^2)^{2-\lambda}}{M_1^2 M_2^2} \exp\left(-\frac{m_s^2+q^2u(1-u)}{M^2}\right)\delta(u-u_0)$$

$M_1^2$ and $M_2^2$ are the Borel parameters associated to the channels $p^2$, $(p+q)^2$ and:

$$M^2 = \frac{M_1^2 M_2^2}{M_1^2+M_2^2}, \quad u_0 = \frac{M_1^2}{M_1^2+M_2^2}$$

The sum rule should be stable under variations of the (unphysical) parameters $M_1^2, M_2^2$




$0.7 \leq M_1^2 \leq 2 \quad GeV^2$

Result: $6.2 \leq g_{f_0 K^+ K^-} \leq 7.8 \quad GeV$

P. Colangelo, F. D.F.
PLB 559 (03) 49

The same method, applied to $g_{\phi K^+ K^-}$ reproduces the experimental datum

Aliev et al., hep-ph/0305056

# SUMMARY OF EXPERIMENTAL DATA

| Collaboration | process | $g_{f_0 K^+ K^-}\,(GeV)$ |
|---|---|---|
| KLOE | $\phi \to f_0\gamma\,(A)$<br>$\phi \to f_0\gamma\,(B)$ | $4.0 \pm 0.2\,(A)$<br>$5.9 \pm 0.1\,(B)$ |
| CMD-2 | $\phi \to f_0\gamma$ | $4.3 \pm 0.5$ |
| SND | $\phi \to f_0\gamma$ | $5.6 \pm 0.8$ |
| WA102 | $pp$ | $2.2 \pm 0.2$ |
| E791 | $D_s \to 3\pi$ | $0.5 \pm 0.6$ |

☆ Fit (A) is without the σ contribution

★
$$\chi^2_{(FIT-A)}/ndf = 109.5/33$$
$$\chi^2_{(FIT-B)}/ndf = 43.2/32$$

# Other theoretical estimates of $g_{f_0 K^+ K^-}$

(Some) older determinations:

- Achasov and Ivanchenko (89) / Achasov and Gubin (97) — according to different scenarios for the structure of $f_0(980)$

  ⇒ $1.95 \leq g_{f_0 K^+ K^-} \leq 7.3 \ \ GeV$

- Nussinov and Truong (89) / Lucio Martinez and Pestieau (90) ⇒ $g_{f_0 K^+ K^-} = 2.74 \ \ GeV$

More recent results:

- Lucio Martinez, Napsuciale (99) (chiral symm. + linear σ model, without mixing with σ ) ⇒ $g_{f_0 K\overline{K}} = \dfrac{M_{f_0}^2 - M_K^2}{2 f_K} \approx 2.24$

- Escribano et al. (02) (from the analysis of the mode $J/\psi \to \phi KK(\pi\pi)$ ) ⇒ $g_{f_0 K^+ K^-} = 2.5 \pm 0.15 \ \ GeV$ (only exp uncertainties)

  $2.0 \pm 0.06 \ \ GeV$

- Oller and Oset (02) ⇒ $g_{f_0 K^+ K^-} = 3.8 \ \ GeV$

# Conclusions

Comparison among different theoretical and experimental determinations show that the value of $g_{f_0 K^+ K^-}$ is rather controversial

From the experimental side, the most accurate determination comes from KLOE, with a measured value larger than other experimental values

The light-cone QCD sum rule result is in keeping with a large value for this parameter, confirming a peculiar aspect of the scalar states, i.e. their large hadronic couplings

⇨ the *hadronic dressing* scenario is favoured

A different determination could be obtained through the analysis of $D_s$ decays to pions and kaons, for example at the B-factories