The $\eta' g^* g^{(*)}$ Vertex Including the η'-Meson Mass

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(Work done in collaboration with Ahmed Ali; hep-ph/0307092)

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Interest in the $\eta' g^* g^{(*)}$ effective vertex function

- The $\eta' g^* g^{(*)}$ effective vertex function $F_{\eta' g^* g^{(*)}}(q_1^2, q_2^2, m_{\eta'}^2)$ (or the η' – gluon transition form factor) enters in a number of decays such as $J/\psi \to \eta'\gamma$, $B \to (\pi, \rho, K, K^*)\eta'$, $B \to \eta' X_s$, $\Upsilon \to \eta' X$, $\Upsilon \to \eta'\gamma$, and hadronic production processes, such as $N + N(\bar{N}) \to \eta' X$, and hence is of great phenomenological importance

- For low gluon virtualities $q_1^2 \sim q_2^2 \ll m_{\eta'}^2$, the vertex is determined by the QCD anomaly

$$F_{\eta' g^* g^{(*)}}(0, 0, m_{\eta'}^2) \equiv F_{\eta' gg}^{\rm A} = -\frac{4\pi\alpha_s(m_{\eta'}^2)}{2\pi^2 f_{\eta'}}$$

- In B- and Υ-meson decays, gluon can have large virtuality, $|q_1^2| \gtrsim m_{\eta'}^2$, in the transition $g^*(q_1) \to g(q_2)\,\eta'(p)$
- Such $\eta' g^* g^{(*)}$ vertices with off-shell gluon(s) can be calculated in perturbative QCD; they involve a convolution of a hard-scattering kernel with the η'-meson wave-function; calculated by several groups

T. Muta & M. Z. Yang, [arXiv:hep-ph/9909484]
A. Ali & A. Y. Parkhomenko, [arXiv:hep-ph/0012212]
P. Kroll & K. Passek-Kumericki, [arXiv:hep-ph/0210045]
S. S. Agaev & N. G. Stefanis, [arXiv:hep-ph/0212318]
A. Ali & A. Y. Parkhomenko, [arXiv:hep-ph/0307092]

- Kagan & Petrov [arXiv:hep-ph/9707354] proposed a phenomenological form for the $\eta' - g$ transition form factor:

$$F_{\eta' g^* g}(q_1^2, 0, m_{\eta'}^2) = \frac{m_{\eta'}^2 H(q_1^2, 0, m_{\eta'}^2)}{q_1^2 - m_{\eta'}^2}$$

  with $H(q_1^2, 0, m_{\eta'}^2) \simeq 1.8 \text{ GeV}^{-1}$ (assumed constant), determined from the $J/\psi \to \eta' \gamma$ decay

- This form is in qualitative agreement with the η'-meson energy spectrum near the upper end in the process $\Upsilon(1S) \to \eta' X$, measured recently by CLEO collaboration [hep-ex/0211029]

- We show that this form of the $\eta' - g$ transition form factor can be obtained in the perturbative hard-scattering approach, if a light-cone wave-function for the η'-meson is used and the η'-meson mass is included

- Remaining uncertainty in the vertex function dominated by non-perturbative parameters, the Gegenbauer coefficients of the η'-meson LCDAs

- Analysis of the data on the $\eta' - \gamma$ transition form factor [P. Kroll & K. Passek-Kumericki, hep-ph/0210045] and the $\Upsilon(1S) \to \eta' X$ decay [Ali & AYP, arXiv:hep-ph/0304278] yields significantly constrained Gegenbauer coefficients, leading to quantitative predictions for the $\eta' - g$ transition form factor, as discussed in this talk

η'-Meson Wave-Function on the Light-Cone

[P. Kroll & K. Passek-Kumericki, hep-ph/0210045]
[A. Ali and A. Parkhomenko, arXiv:hep-ph/0307092]

- η'-meson contains both quark-antiquark and gluonic components
- Work in the leading-twist (twist-two) approximation of the η'-meson

Quark-Antiquark Component

- Quark-antiquark component of the η'-meson is interpolated by the bilocal axial-vector operator in the $SU(3)$ flavour-singlet state

$$\mathcal{O}_{5\mu}^{(q)}(x, y) = \frac{1}{\sqrt{N_f}} \bar{\Psi}(x) \gamma_\mu \gamma_5 \, [x, y] \, \Psi(y),$$

where $\Psi(x) = (u(x), d(x), s(x))$ and $N_f = 3$. (Summation over the Dirac, colour and flavour indices is implicitly assumed)

- The decay constant $f_{\eta'}$ is determined in the local limit of the above operator

$$\langle 0 | \mathcal{O}_{5\mu}^{(q)}(0, 0) | \eta'(p) \rangle = \langle 0 | \frac{1}{\sqrt{N_f}} \bar{\Psi}(0) \gamma_\mu \gamma_5 \Psi(0) | \eta'(p) \rangle = i f_{\eta'} p_\mu,$$

- The quark-antiquark twist-two LCDA $\phi_{\eta'}^{(q)}(u)$

$$i f_{\eta'} \, \phi_{\eta'}^{(q)}(u) = 2 \int \frac{dz^-}{2\pi} \, e^{-i\xi(Pz)} \langle 0 | n^\mu \mathcal{O}_{5\mu}^{(q)}(z, -z) | \eta'(p) \rangle$$

Here, $\xi = 2u - 1$, u and $1 - u$ are the scaled energies of the partons inside the η'-meson, and the separation between the quark fields in the operator is assumed to be light-like ($z^2 = 0$ and $z_\mu = z^- \, n_\mu$).

- Normalization and symmetry condition:

$$\int_0^1 du\, \phi_{\eta'}^{(q)}(u) = 1, \qquad \phi_{\eta'}^{(q)}(u) = \phi_{\eta'}^{(q)}(\bar{u})$$

- η'-meson mass is taken into account, $p^2 = m_{\eta'}^2$

$$P_\mu = p_\mu - \frac{m_{\eta'}^2 z_\mu}{2(pz)}, \qquad (Pz) = (pz)$$

- In the momentum space,

$$\mathcal{P}^{(q)}_{j\beta b;i\alpha a} = \frac{1}{4N_c}\,[\gamma_5(P\gamma)]_{ji}\,\delta_{\beta\alpha}\,\frac{\delta_{ba}}{\sqrt{N_f}}$$

 can be interpreted as the η'-meson light-cone projection operator onto the state of the incoming quark and antiquark

Gluonic Component

- Partially traceless and symmetric bilocal gluonic operator interpolates the gluonic content of the η'-meson

$$\tilde{\mathcal{O}}^{(g)}_{\mu\nu}(x,y) = \frac{1}{2}\Big[G_{\mu\alpha}(x)[x,y]\tilde{G}_{\nu}{}^{\alpha}(y) + G_{\nu\alpha}(x)[x,y]\tilde{G}_{\mu}{}^{\alpha}(y)\Big]$$

$$- \frac{g_{\mu\nu}}{4} G_{\alpha\beta}(x)[x,y]\tilde{G}^{\alpha\beta}(y)$$

- Gluonic LCDA, $\phi^{(g)}_{\eta'}(u)$, is defined by the matrix element with a light-like separation $z^2 = 0$ between gluonic fields

$$\langle 0|\tilde{\mathcal{O}}^{(g)}_{\mu\nu}(z,-z)|\eta'(p)\rangle = \frac{f_{\eta'} C_F}{2\sqrt{N_f}} P_\mu P_\nu \int_0^1 du\, e^{i\xi(pz)} \phi^{(g)}_{\eta'}(u)$$

- In the local limit, the bilocal gluonic operator vanishes $[\tilde{\mathcal{O}}^{(g)}_{\mu\nu}(0,0) = 0]$

$$G_{\mu\alpha}(0)\tilde{G}_{\nu}{}^{\alpha}(0) = \frac{g_{\mu\nu}}{4} G_{\alpha\beta}(0)\tilde{G}^{\alpha\beta}(0)$$

Normalization and symmetry condition for the gluonic LCDA:

$$\int_0^1 du\, \phi^{(g)}_{\eta'}(u) = 0 \qquad \phi^{(g)}_{\eta'}(u) = -\phi^{(g)}_{\eta'}(\bar{u})$$

- Normalization leaves an arbitrariness in the choice of a constant prefactor in the matrix element. The choice made here is determined by specific forms of the non-diagonal anomalous dimensions

- The usual Feynman rules involve the gluonic four-potential $A_\mu^A(z)$ instead of the gluonic field strength tensor $G_{\mu\nu}^A(z)$; the light-cone gauge, $n^\mu A_\mu^A(x;n) = 0$, is used for getting the required matrix element:

$$\langle 0 \big| A^A_{[\mu}(z) A^B_{\nu]}(-z) \big| \eta'(p) \rangle = \frac{f_{\eta'} C_F}{4\sqrt{N_f}} \frac{\delta_{AB}}{2 N_c C_F}$$

$$\times \frac{\varepsilon_{\mu\nu\rho\sigma} z^\rho P^\sigma}{(zP)} \int\limits_0^1 du\, e^{i\xi(pz)} \frac{\phi^{(g)}_{\eta'}(u)}{u\bar{u}}$$

Here, $A^A_{[\mu}(x) A^B_{\nu]}(y) \equiv [A^A_\mu(x) A^B_\nu(y) - A^A_\nu(x) A^B_\mu(y)]/2$ is the bilocal operator antisymmetrized in the Lorentz indices

- In the momentum space,

$$\mathcal{P}^{(g)}_{\mu A;\nu B} = \frac{i\,\delta_{AB}}{4 N_c \sqrt{N_f}} \frac{\varepsilon_{\mu\nu\rho\sigma} n^\rho p^\sigma}{(np)}$$

is the projection operator of the η'-meson onto the state of two incoming gluons

Light-Cone Distribution Amplitudes

- The leading-twist (twist-two) LCDAs of the η'-meson

$$\phi_{\eta'}^{(q)}(u,Q^2) = 6u\bar{u}\left[1 + \sum_{\text{even}\, n\geq 2} A_n(Q^2)\, C_n^{3/2}(u-\bar{u})\right]$$

$$\phi_{\eta'}^{(g)}(u,Q^2) = u^2\bar{u}^2 \sum_{\text{even}\, n\geq 2} B_n(Q^2)\, C_{n-1}^{5/2}(u-\bar{u})$$

- Gegenbauer moments

$$A_n(Q^2) = B_n^{(q)}(\mu_0^2)\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^n} + \rho_n^{(g)}\, B_n^{(g)}(\mu_0^2)\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^n}$$

$$B_n(Q^2) = \rho_n^{(q)}\, B_n^{(q)}(\mu_0^2)\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_+^n} + B_n^{(g)}(\mu_0^2)\left[\frac{\alpha_s(\mu_0^2)}{\alpha_s(Q^2)}\right]^{\gamma_-^n}$$

- Quantities completely defined by the perturbative QCD

$$\gamma_\pm^n \equiv \frac{1}{2}\left[\gamma_{QQ}^n + \gamma_{GG}^n \pm \sqrt{(\gamma_{QQ}^n - \gamma_{GG}^n)^2 + 4\gamma_{QG}^n\gamma_{GQ}^n}\,\right]$$

$$\rho_n^{(q)} = 6\,\frac{\gamma_+^n - \gamma_{QQ}^n}{\gamma_{QG}^n}, \qquad \rho_n^{(g)} = \frac{1}{6}\,\frac{\gamma_{QG}^n}{\gamma_-^n - \gamma_{QQ}^n}$$

- The anomalous dimensions γ_{ij}^n:

$$\gamma_{QQ}^n = \frac{C_F}{\beta_0}\left[3 + \frac{2}{(n+1)(n+2)} - 4\sum_{j=1}^{n+1}\frac{1}{j}\right]$$

$$\gamma_{QG}^n = \frac{C_F}{\beta_0}\frac{n(n+3)}{3(n+1)(n+2)} \qquad \gamma_{GQ}^n = \frac{N_f}{\beta_0}\frac{12}{(n+1)(n+2)}$$

$$\gamma_{GG}^n = \frac{N_c}{\beta_0}\left[\frac{8}{(n+1)(n+2)} - 4\sum_{j=1}^{n+1}\frac{1}{j}\right] + 1$$

Here, $\beta_0 = (11N_c - 2n_f)/3$ and n_f is the number of quarks with masses less than the energy scale Q entering in the LCDAs

- Approximate forms for the η'-meson LCDAs in which only the second Gegenbauer moments ($n = 2$) are kept:

$$\phi_{\eta'}^{(q)}(u, Q^2) = 6u\bar{u}\left[1 + 6(1 - 5u\bar{u})\, A_2(Q^2)\right]$$

$$\phi_{\eta'}^{(g)}(u, Q^2) = 5u^2\bar{u}^2\,(u - \bar{u})\, B_2(Q^2)$$

$\eta' g^* g^{(*)}$ Effective Vertex Function

- In the momentum space, the $\eta' g^* g^*$ vertex can be extracted from the invariant matrix element of the process $\eta'(p) \to g^*(q_1)\, g^*(q_2)$

$$\mathcal{M} = \mathcal{M}^{(q)} + \mathcal{M}^{(g)} \equiv F_{\eta' g^* g^*}(q_1^2, q_2^2, m_{\eta'}^2)\, \delta_{AB}\, \varepsilon^{\mu\nu\rho\sigma}\, \varepsilon_{1\mu}^{A*} \varepsilon_{2\nu}^{B*} q_{1\rho} q_{2\sigma}$$

- Individual quark-antiquark and gluonic contributions

$$\mathcal{M}^{(q)} = i f_{\eta'} \int_0^1 du\, \phi_{\eta'}^{(q)}(u, Q^2)\, \mathcal{P}^{(q)}_{j\beta b;i\alpha a}\, \delta^{ab} \left[T_{\mathrm{H}}^{(q)}\right]_{ij}^{\alpha\beta}$$

$$\mathcal{M}^{(g)} = \frac{i f_{\eta'}}{2} \int_0^1 du\, \frac{\phi_{\eta'}^{(g)}(u, Q^2)}{u\bar{u}}\, \mathcal{P}^{(g)}_{\sigma D;\rho C} \left[T_{\mathrm{H}}^{(g)}\right]_{CD}^{\rho\sigma}$$

Here, $\left[T_{\mathrm{H}}^{(q)}\right]_{ij}^{\alpha\beta}$ and $\left[T_{\mathrm{H}}^{(g)}\right]_{CD}^{\rho\sigma}$ are the quark and gluonic hard-scattering kernels calculated in the perturbative QCD

- Kinematical quantities defined as follows:

$$q^2 = q_1^2 + q_2^2 \qquad \omega = \frac{q_1^2 - q_2^2}{q^2} \qquad \eta = \frac{m_{\eta'}^2}{q^2}$$

- Light-like vector n_μ; should be related with four-momenta of gluons

$$n_\mu = \tilde{C}_n \left[(1-\omega)\, p_\mu + (\omega - \eta + \omega\lambda)\, q_{2\mu}\right]$$

$\tilde{C}_n$ is an arbitrary parameter

$$\lambda = \sqrt{1 - \frac{2\eta}{\omega^2} + \frac{\eta^2}{\omega^2}}$$

The quark-antiquark and gluonic projection operators in terms of the gluon four-momenta

$$\mathcal{P}^{(q)}_{j\beta b;i\alpha a} = \frac{\delta_{\beta\alpha}}{4N_c}\frac{\delta_{ba}}{\sqrt{N_f}}\frac{1}{2\omega\lambda}\left[\gamma_5\left\{(\omega-\eta+\omega\lambda)(p\gamma)+2\eta\,(q_2\gamma)\right\}\right]_{ji}$$

$$\mathcal{P}^{(g)}_{\mu A;\nu B} = \frac{i\,\delta_{AB}}{4N_c\sqrt{N_f}}\frac{2}{\omega\lambda}\frac{\varepsilon_{\mu\nu\rho\sigma}q_1^\rho q_2^\sigma}{q^2}$$

- Both operators contain $1/\lambda$:

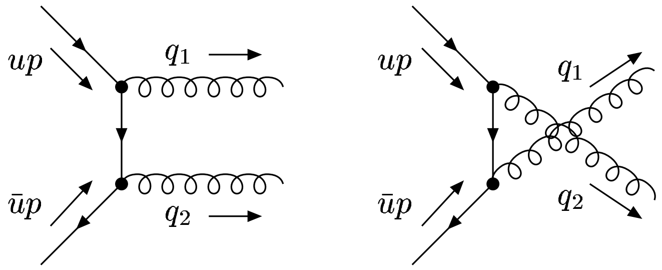
$$1/\lambda = 1 \qquad \text{when} \qquad m_{\eta'} = 0$$

$$1/\lambda = m^2_{\eta'}/(q_1^2 - m^2_{\eta'}) \qquad \text{when} \qquad q_2^2 = 0$$

Phenomenological form for the $\eta' - g$ transition form factor, suggested by Kagan & Petrov, is reproduced in this approach

- The appearance of the singularity at $q_1^2 = m^2_{\eta'}$, when one of the gluons is on-shell, results from the light-cone form of the η'-meson wave-function adopted here; Inclusion of the transverse degrees of freedom for the partons inside the η'-meson removes this unphysical singularity

Quark Part of the $\eta' g^* g^*$ Vertex



- General expression for the quark part of the $\eta' g^* g^*$ vertex

$$F^{(q)}_{\eta' g^* g^*}(q^2, \omega, \eta) = \frac{4\pi\alpha_s(Q^2)}{m^2_{\eta'}\lambda} \frac{3 f_{\eta'}\sqrt{N_f}}{N_c}$$

$$\times \left\{ G_0^{(q)}(\omega, \eta) + 6 A_2(Q^2) G_2^{(q)}(\omega, \eta) \right\}$$

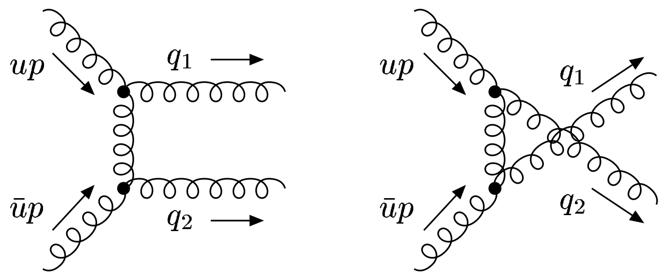
Functions $G_0^{(q)}(\omega, \eta)$ and $G_2^{(q)}(\omega, \eta)$ can be found in hep-ph/0307092

- When $m_{\eta'} = 0$, the usual $1/q^2$ behaviour is reproduced

$$F^{(q)}_{\eta' g^* g^*}(q^2, \omega, 0) = \frac{4\pi\alpha_s(|q^2|)}{q^2} \frac{3 f_{\eta'}\sqrt{N_f}}{N_c}$$

$$\times \left\{ f_0(\omega) + 6 A_2(|q^2|) f_2(\omega) \right\}$$

Gluonic Part of the $\eta' g^* g^*$ Vertex



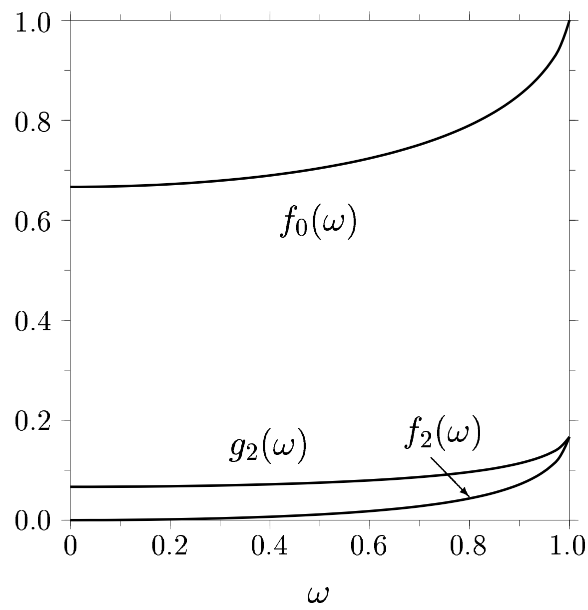
- General expression for the gluonic part of the $\eta' g^* g^*$ vertex

$$F^{(g)}_{\eta' g^* g^*}(q^2, \omega, \eta) = -\frac{4\pi\alpha_s(Q^2)}{m^2_{\eta'}\lambda} \frac{5 f_{\eta'}}{2\sqrt{N_f}} B_2(Q^2)\, \mathrm{G}^{(g)}_2(\omega, \eta)$$

  For the explicit form of $\mathrm{G}^{(g)}_2(\omega, \eta)$ we refer to hep-ph/0307092

- In the massless limit of the η'-meson

$$F^{(g)}_{\eta' g^* g^*}(q^2, \omega, 0) = -\frac{4\pi\alpha_s(|q^2|)}{q^2} \frac{5 f_{\eta'}}{2\sqrt{N_f}} B_2(|q^2|)\, g_2(\omega),$$



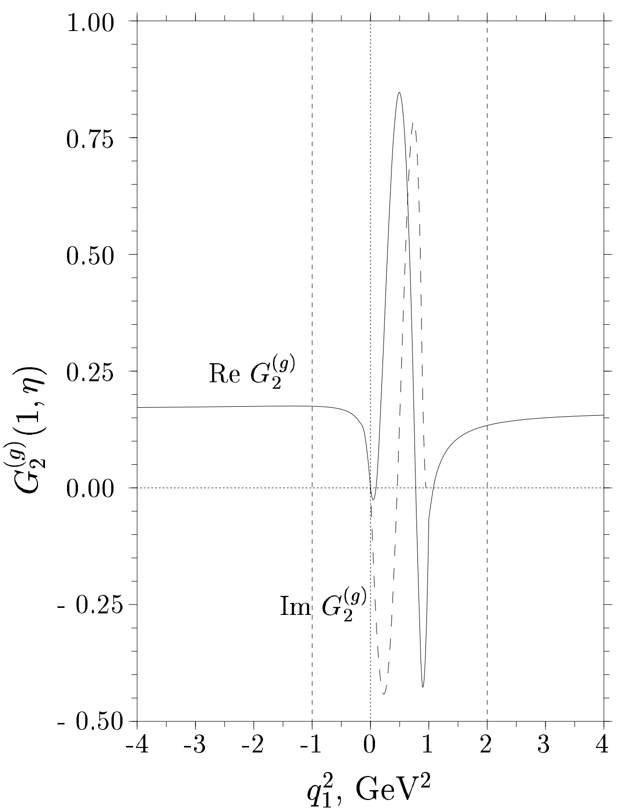
$\eta' - g$ Transition Form Factor

- In the light-cone model for the η'-meson wave-function, the $\eta' - g$ transition form factor has the form:

$$F_{\eta' g}(q_1^2) = \frac{m_{\eta'}^2 \, H(q_1^2)}{q_1^2 - m_{\eta'}^2}$$

- Perturbative QCD analysis determines the q_1^2 dependence of $H(q_1^2)$:

$$H(q_1^2) = \frac{4\pi\alpha_s(Q^2)}{m_{\eta'}^2} \sqrt{3} f_{\eta'} \left[1 + A_2(Q^2) - \frac{5}{6} B_2(Q^2) G_2^{(g)}(1,\eta) \right]$$



- $G_2^{(g)}(1,\eta) \;\to\; 1/6$; it is a good approximation in the regions of applicability of the perturbative QCD: $q_1^2 < -1$ GeV^2 and $q_1^2 > 2$ GeV^2

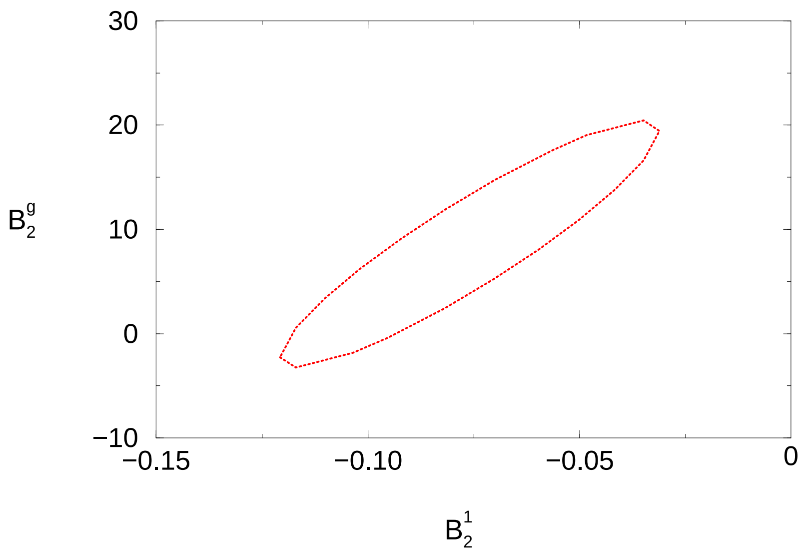
Constraints on the Parameters of LCDAs

- A fit to the CLEO and L3 data on the $\eta' - \gamma$ transition form factor for $Q^2 > 2$ GeV^2 was recently undertaken by Kroll & Passek-Kumericki [hep-ph/0210045]

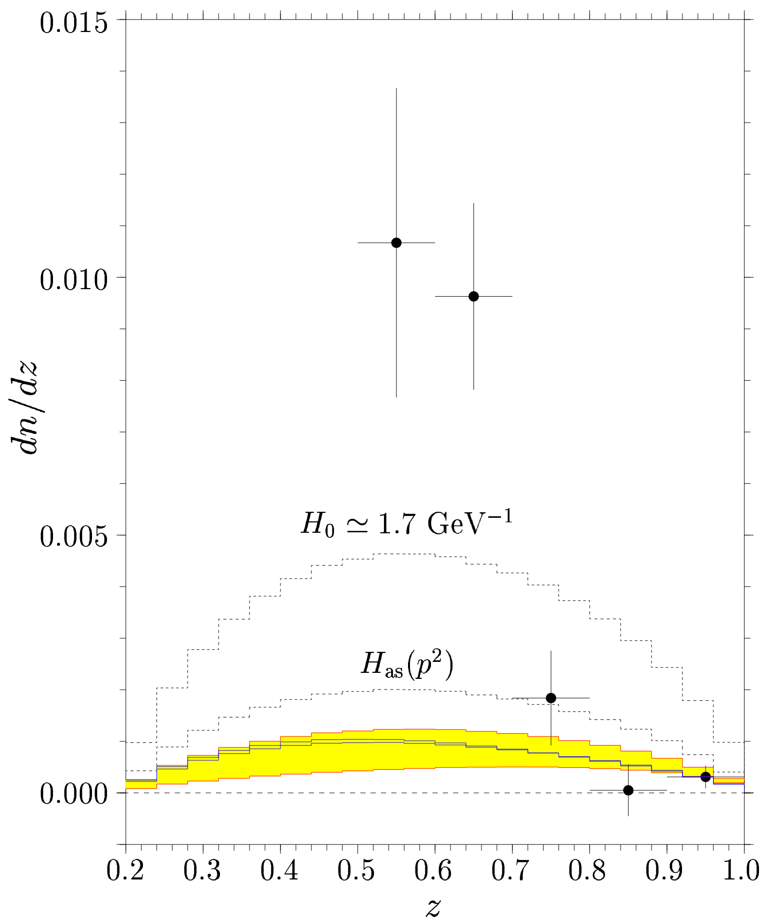
$$A_2(1\ \text{GeV}^2) = -0.08 \pm 0.04 \qquad B_2(1\ \text{GeV}^2) = 9 \pm 12$$

$$B_2^{(q)}(1\ \text{GeV}^2) = 0.02 \pm 0.17 \qquad B_2^{(g)}(1\ \text{GeV}^2) = 9.0 \pm 11.5$$

- This leaves an order of magnitude uncertainty on both the gluonic Gegenbauer moment and Gegenbauer coefficient
- Strong correlation between the Gegenbauer moments was observed



- The other process which allows to get an independent information on the Gegenbauer coefficients in the $\eta' g^* g$ vertex is the inclusive $\Upsilon(1S) \to \eta' X$ decay
- The η'-meson energy spectrum was recently measured by the CLEO collaboration [hep-ex/0211029]

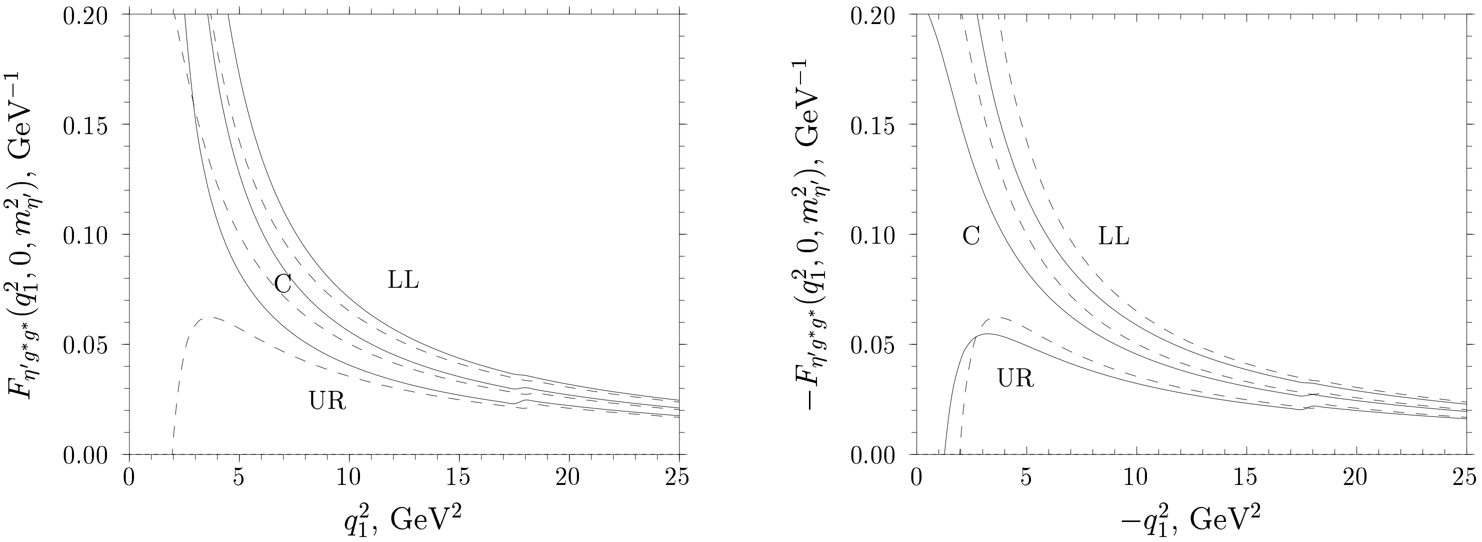


- In the hard part of the spectrum, the data are in agreement with the perturbative QCD analysis
- Combined best fit on the data from the $\eta' - \gamma$ transition form factor and the $\Upsilon(1S) \to \eta' X$ decay results:

$$B_2^{(q)}(2\ \mathrm{GeV}^2) = -0.008 \pm 0.054 \quad B_2^{(g)}(2\ \mathrm{GeV}^2) = 4.6 \pm 2.5$$

$$A_2(2\ \mathrm{GeV}^2) = -0.054 \pm 0.029 \quad B_2(2\ \mathrm{GeV}^2) = 4.6 \pm 2.7$$

η'-Meson Mass Effect in the $\eta' g^* g$ Vertex



- Inclusion of the η'-meson mass reduces the parameteric dependence on the Gegenbauer coefficients of the $\eta' g^* g$ effective vertex function in the time-like region of the gluon virtuality
- This is generally not the case for the space-like gluon virtuality, in particular for low values of the virtuality, as the η'-meson mass effects are not as pronounced
- The pert. calculated $\eta' g^* g$ vertex is a smooth function in the space-like region, but, with the Gegenbauer coefficients determined from data, there exists a marked mismatch between the extrapolated vertex function and the anomaly normalization at $q_1^2 = 0$. Thus, non-perturbative effects are crucial in the low-$|q_1^2|$ region
- A phenomenological interpolation formula for the effective vertex function is proposed for space-like region, reproducing the anomaly normalization at $q_1^2 = 0$, and the perturbative QCD bahaviour for large $|q_1^2|$

Interpolating Formula for Space-Like Virtuality

- The anomaly value for the transition form factor

$$F^{\rm A}_{\eta' gg} = -4\pi\alpha_s(m^2_{\eta'})\,\frac{1}{2\pi^2 f_{\eta'}} = -H_{\rm A}$$

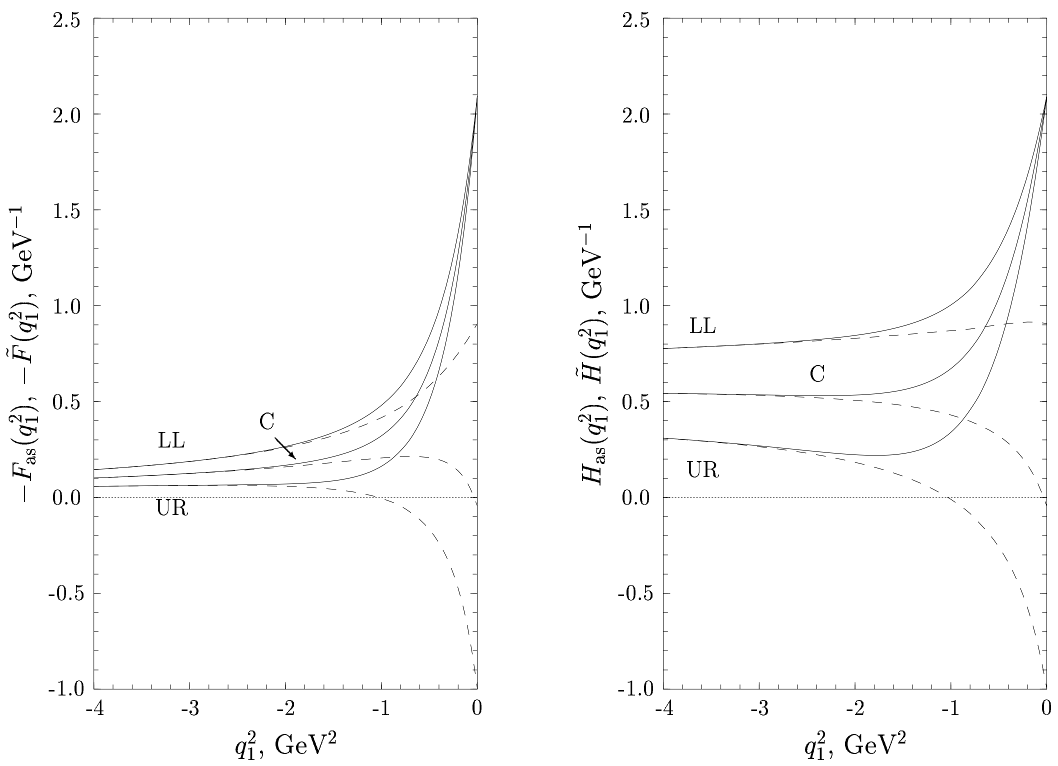
- The form in the large q_1^2 asymptotic from the perturbative QCD

$$H_{\rm as}(q_1^2) = \frac{4\pi\alpha_s(Q^2)}{m^2_{\eta'}}\,\sqrt{3}f_{\eta'}\left[1 + A_2(Q^2) - \frac{5}{36}\,B_2(Q^2)\right]$$

The dependence on q_1^2 is coming only through $Q^2 = |q_1^2| + m^2_{\eta'}$

- A *formal* limit of the function $H_{\rm as}(q_1^2)$ for on-shell gluons ($q_1^2 = 0$) exists with a strong dependence on the Gegenbauer coefficients
- Modification of the perturbative result ($C_s = 2$)

$$\tilde{H}(q_1^2) = H_{\rm as}(q_1^2) + [H_A - H_{\rm as}(0)]\,\exp\left[C_s\, q_1^2/m^2_{\eta'}\right]$$



Summary

- The $\eta' g^* g^{(*)}$ effective vertex function is calculated in the perturbative QCD approach using the light-cone DAs for the η'-meson with the inclusion of the η'-meson mass
- If one of the gluons is on the mass shell, the pole-like behaviour of the η' – gluon transition form factor emerges in this approach for both the quark-antiquark and the gluonic parts of the form factor due to the light-cone model for the η'-meson wave-function. The corresponding function $H(q_1^2, 0, m_{\eta'}^2)$ is perturbatively calculated
- The Gegenbauer coefficients, required for a quantitative analysis of the vertex function, are determined from the combined analysis of the $\eta' - \gamma$ transition form factor and the $\Upsilon(1S) \to \eta' X$ decay
- The corrections due to the η'-meson mass are analyzed numerically, with the result that they are important for lower values of the gluon virtuality, in particular in the time-like region
- An interpolating formula connecting the QCD anomaly value and the perturbative-QCD behaviour of the η' – gluon transition form factor is presented for the space-like gluon virtuality, taking into account the η'-meson mass, which modifies the vertex function significantly in the region $|q_1^2| < 1$ GeV^2 and reduces the theoretical dispersion in low $|q_1^2|$ region considerably