HEP2003 Aachen, Session *'Tests of the Standard Model'* 17 July 2003

Theoretical evaluations of the running α and $g-2$ of the muon

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I. $(g-2)_\mu$: Introduction; recent developments; latest news

II. $\Delta\alpha_{\rm QED}^{\rm had}$: Status; new evaluation from HMNT

III. Outlook: Future improvements / changes

I. $(g-2)_\mu$

- Magnetic moment $\vec{\mu}$: a fundamental observable

$$\vec{\mu} = -g \frac{e}{2m} \vec{s}, \qquad \vec{s} = \frac{1}{2} \vec{\sigma} \quad \text{the particle's spin.}$$

- Dirac equation $\rightsquigarrow$ $g = 2$, but quantum corrections lead to the anomalous magnetic moment: $a \equiv (g-2)/2$

- Theory prediction in QED at one loop:

$$a = \alpha/(2\pi) \simeq 11\,614\,097 \cdot 10^{-10} \qquad \text{(Schwinger)}$$

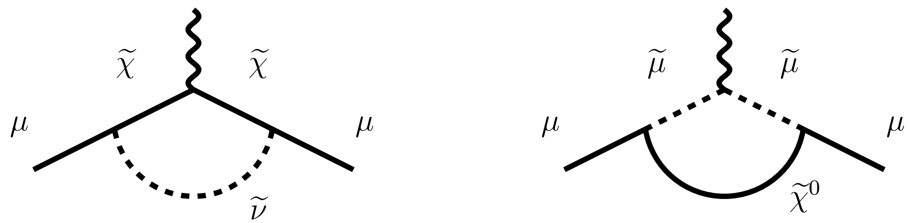
- Experimentally: one of the most precisely measured quantities

$$a_\mu^{\rm exp} = 11\,659\,203\,(8) \cdot 10^{-10} \quad [0.7\,\text{ppm}]$$
$$a_e^{\rm exp} = 11\,596\,521.869\,(0.041) \cdot 10^{-10} \quad [3.5\,\text{ppb}]$$

- a_e 200 times more precise than a_μ, but: Sensitivity to New Physics typically enhanced by $m_\mu^2/m_e^2 \sim 43000$ $\qquad (a^{\rm NP} \propto m^2/\Lambda_{\rm NP}^2)$

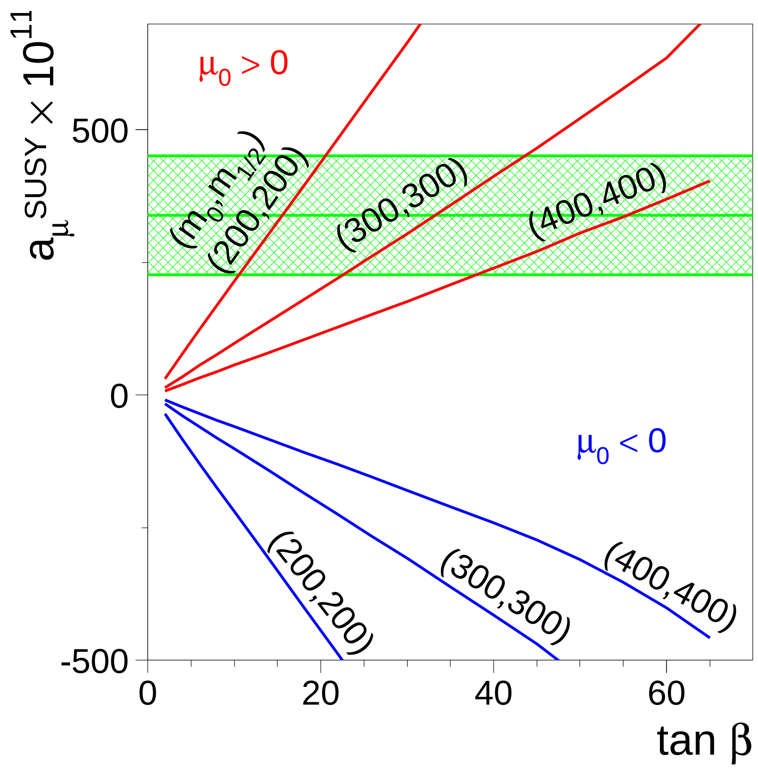
- Example: SUSY contributions in a_μ?

They come mainly from:



In order to be $14 \leq a_\mu^{\rm SUSY} \cdot 10^{10} \leq 58$ (2 σ range),

$\rightsquigarrow$ $\tilde{m} = 150 - 670$ GeV for $\tan\beta = 10 - 50$.



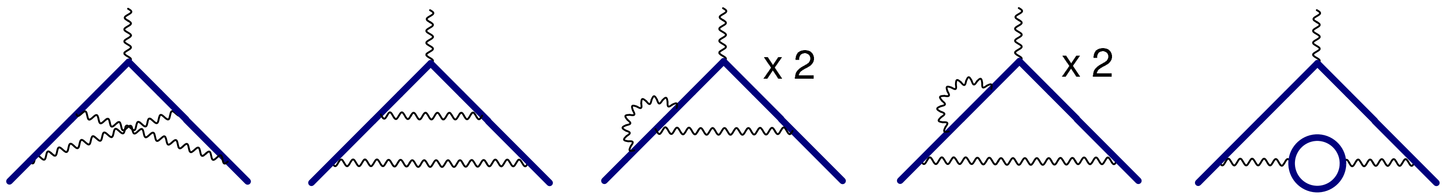
$\Delta a_\mu = (339 \pm 112) \cdot 10^{-11}$

figure from W. de Boer + C. Sander, `hep-ph/0307049`

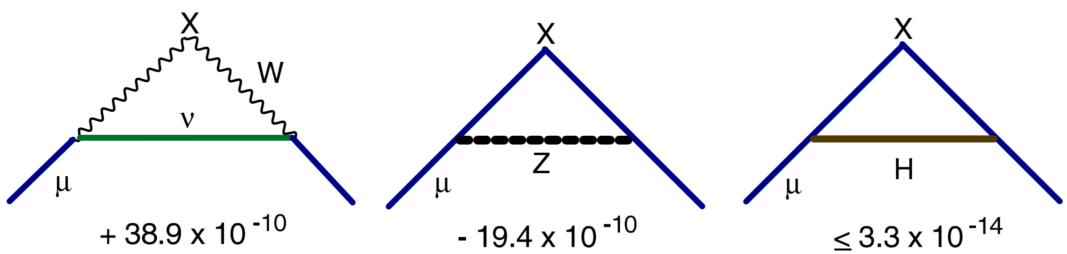
The different contributions in the SM: $a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$

Some typical Feynman graphs for illustration:

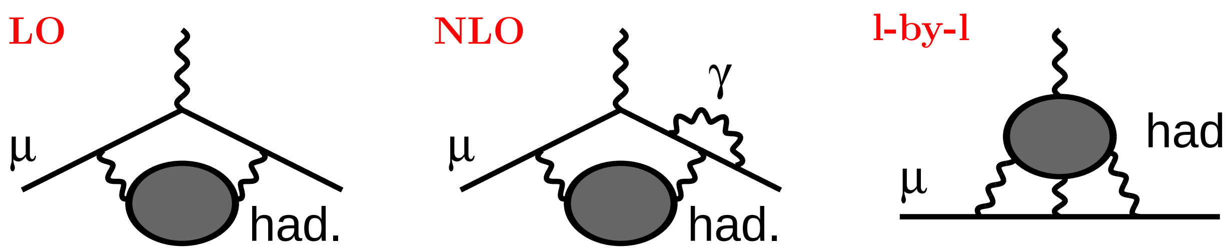
QED 2–loop:



EW 1–loop:



Hadronic contributions: $a_\mu^{\rm had} = a_\mu^{\rm had,LO} + a_\mu^{\rm had,NLO} + a_\mu^{\rm had,L-by-L}$

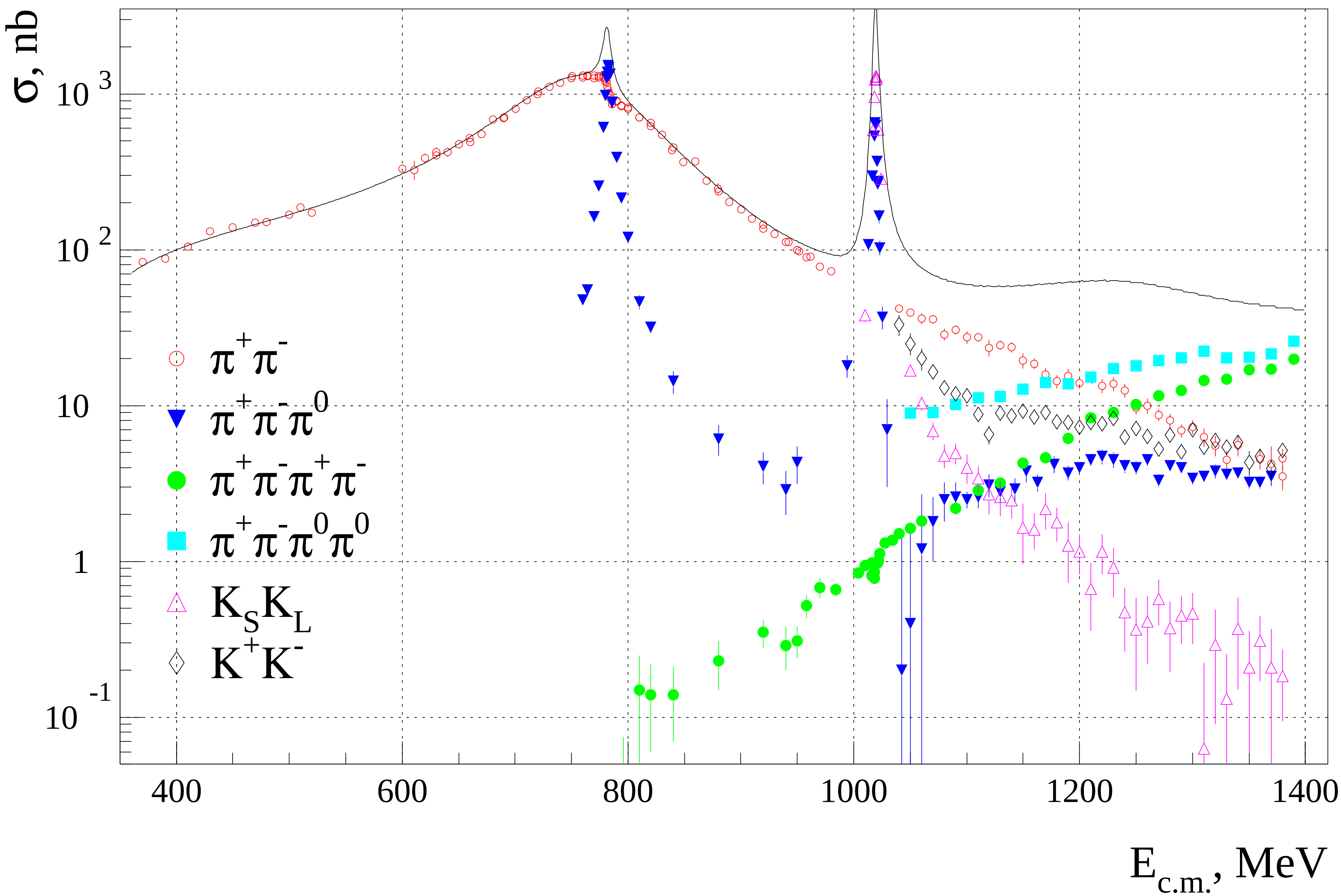


$$a_\mu^{\rm had,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} {\rm d}s\, \sigma^0_{\rm had}(s) K(s)\,, \quad \text{with } K(s) = \frac{m_\mu^2}{3s}\cdot(0.63\ldots 1)$$

Example for $\sigma_{\rm had}$ data 'input':

Figure from Simon Eidelman

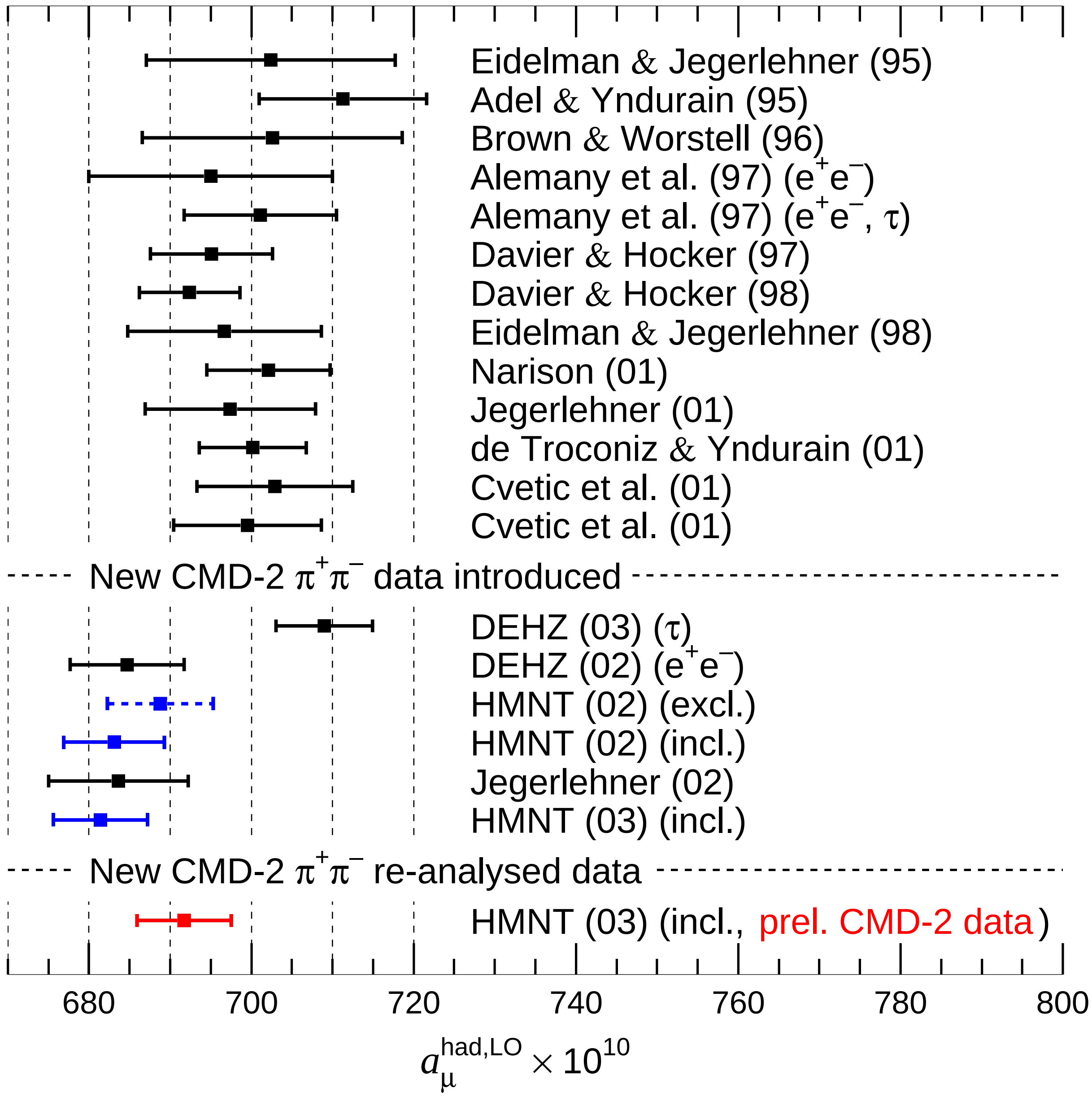
CMD-2 $\sigma(e^+e^- \to hadrons)$ measurements in various channels:



The different contributions numerically:

Source	contr. to $a_\mu \times 10^{11}$	remarks
QED	$116\ 584\ 703.5 \pm 2.8$	up to 5-loop! (Laporta+Remiddi, Kinoshita et al.)
	(was $116\ 584\ 705.7 \pm 2.9$)	▶ incl. recent correction from Kinoshita+Nio
EW	154 ± 2	2-loop, Czarnecki+Marciano+Vainshtein
		▶ agrees very well with Knecht+Peris+Perrottet+de Rafael
LO hadr.	$7090 \pm 51 \pm 12 \pm 28$	Davier+Eidelman+Hoecker+Zhang (τ)
	$6847 \pm 60 \pm 36$	Davier+Eidelman+Hoecker+Zhang (e^+e^-)
	$6917 \pm 58 \pm 20$	Hagiwara+Martin+Nomura+T, w. prel. CMD-2 $\pi\pi$ re-anal.
NLO hadr.	-100 ± 6	Alemany+Davier+Hoecker '98 in agreem. with Krause '97
L-by-L	80 ± 40	compilation from Nyffeler, `hep-ph/0203243`
< Nov. 2001:	(-85 ± 25)	the 'famous' sign error, $2.6\,\sigma \longrightarrow 1.6\,\sigma$
$\sum$	$(11659175.5 \pm 7.4)10^{-10}$	with prel. HMNT (using e^+e^-)
Exp.:	$(11\ 659\ 203 \pm 8) \cdot 10^{-10}$	BNL E821 world average 2002
$a_\mu^{EXP} - a_\mu^{TH}$	$(27.5 \pm 11) \cdot 10^{-10}$	$\sim 2.5\,\sigma$ with e^+e^-, $0.9\,\sigma$ using τ (DEHZ)

Different eval. of $a_\mu^{\text{had, LO}}$; new HMNT with prel. CMD-2 $\pi^+\pi^-$ re-analysis

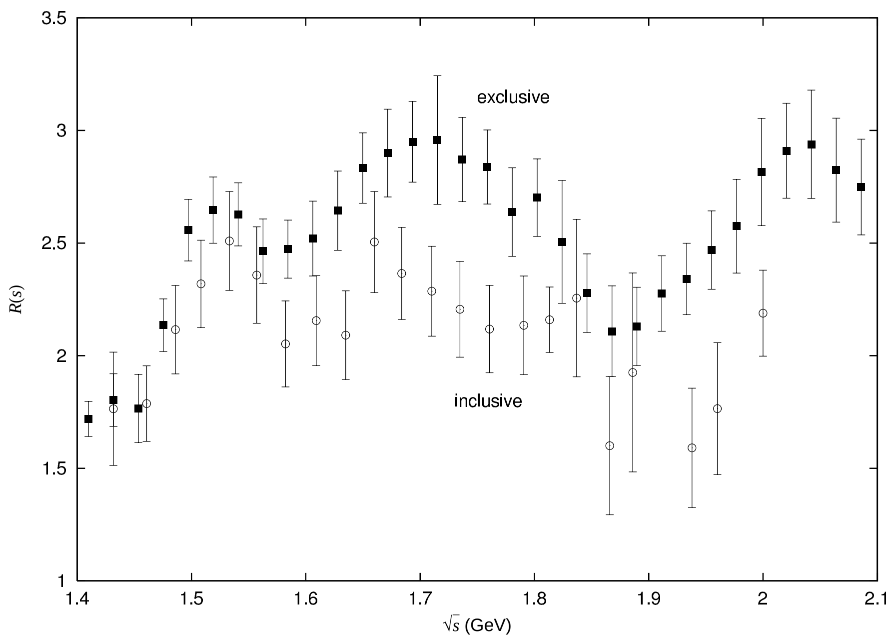


Features of the HMNT-analysis:

- e^+e^- data driven. Very complete data collection.
- Data 'clustering' using non-linear χ^2 fit makes best use of stat. and sys. error information.
- Straightforward $\int$ over combined data for all energies (even over ω and ϕ resonances).
- Correlations between different energies.

'Puzzle' excl. vs. incl. (a pure data issue?)

Incl. 'selected' by QCD Sum Rule analysis.

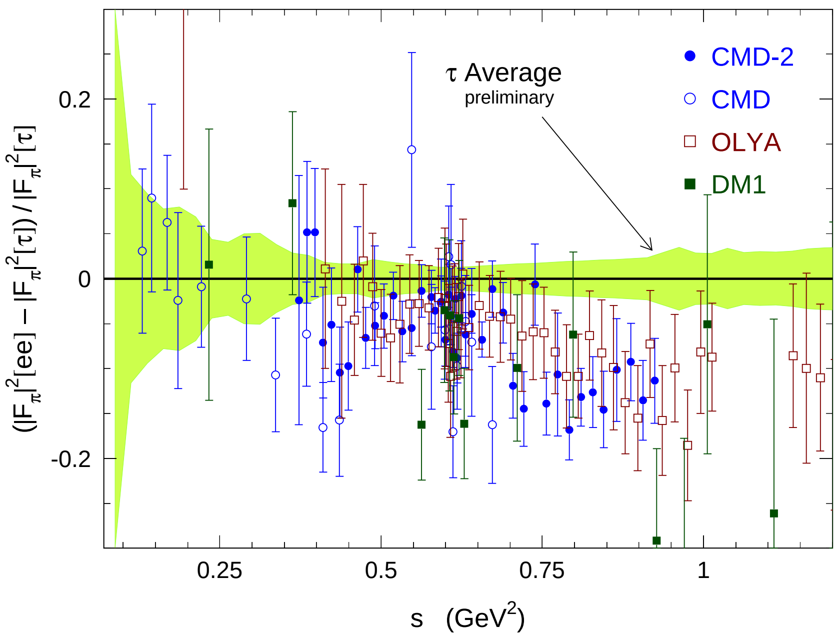


Incl. and $\sum$ excl. data after clustering and fitting

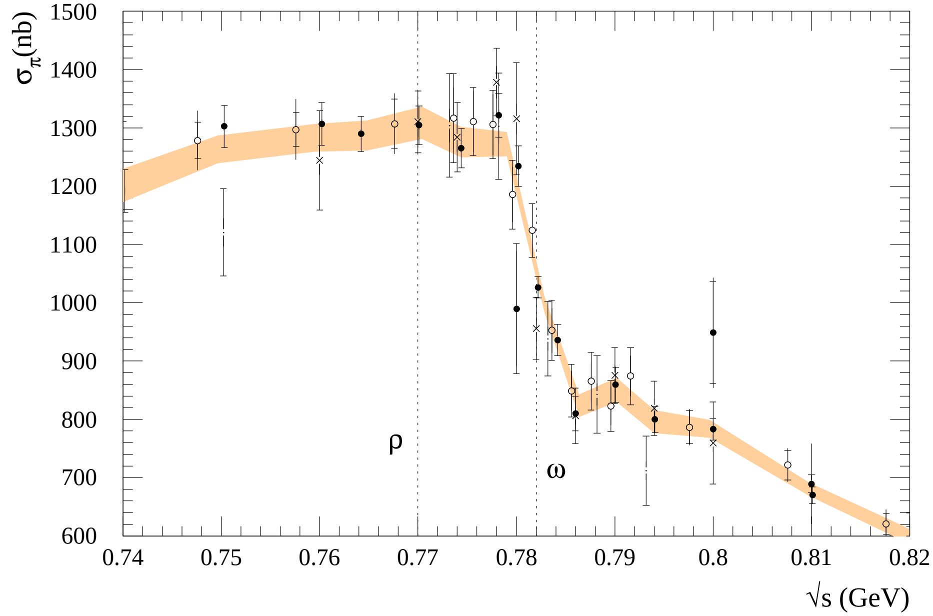
Contributions from the different regions

energy range	comments	$a_\mu^{\text{had, LO}} \times 10^{10}$
$2m_\pi \ldots 0.32$	chiral PT	2.30 ± 0.05
$0.32 \ldots 1.43$	excl. only	605.40 ± 5.15
$1.43 \ldots 2.00$	excl. only	35.98 ± 1.68
	incl. only	32.41 ± 2.46
$2.00 \ldots 11.09$	incl. only	42.12 ± 1.14
$J/\psi + \psi(2S)$	NW	7.31 ± 0.43
$\Upsilon(1-6S)$	NW	0.10 ± 0.00
$11.09 \ldots \infty$	pQCD	2.14 ± 0.01
$\sum$ of all	'excl.'	695.35 ± 5.61
	'incl.'	691.78 ± 5.84

DEHZ (03): e^+e^- compared to τ data:



'Zoom': $\rho - \omega$ interference (HMNT):



- Important improvements recently came through new data from Beijing (BES, inclusive) and from Novosibirsk (CMD-2 (0.6% sys. err. for $\pi\pi$!) and SND, exclusive channels).

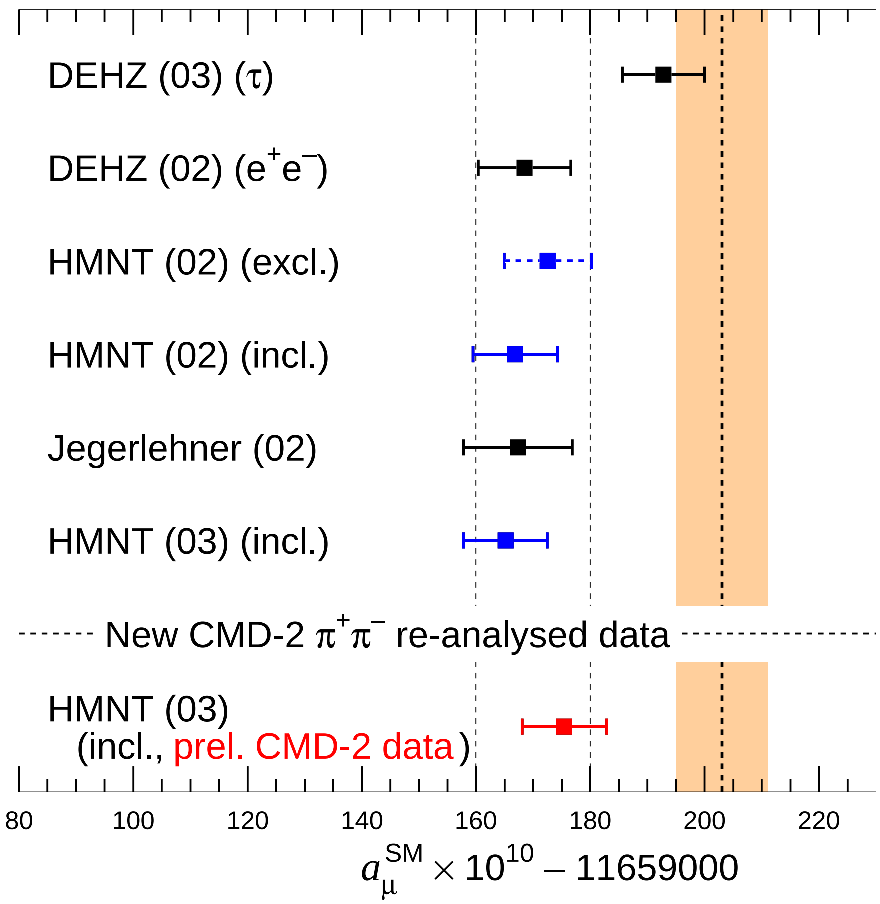
▶ New unresolved puzzle: Why are τ spectral function data not compatible with e^+e^- ?

!Connection of $\tau^- \to \nu_\tau \pi^- \pi^0, \nu_\tau \pi^- 3\pi^0, \nu_\tau 2\pi^- \pi^+ \pi^0$ and $e^+e^- \to n\,\pi's$ not direct:

- Violation of CVC hypothesis (Isospin-symmetry) $\rightsquigarrow$ complicated corrs. ($\to$ Cirigliano+Ecker+Neufeld) due to short+long distance rad. corrs., mass differences ($m_{\pi^-} \neq m_{\pi^0}$ etc.), $\rho - \omega$ interference, ...
- Is everything under control *at the % level of accuracy*? Is something wrong with data?

$\longrightarrow$ independent check: KLOE Radiative Return analysis agrees with e^+e^- (see next talk).

$a_\mu^{\rm SM}$ compared to the BNL 02 world av.



Th. and Exp. accuracy comparable!

Is soon Light-by-Light limiting?!

Summary (g-2):

- τ 'puzzle' remains unresolved.
- Preliminary results from a CMD-2 re-analysis ($\pi^+\pi^-$ data) move a_μ slightly upwards; a discrepancy of $\sim 2.x\,\sigma$ remains!
- Recent e^+e^- based evaluations agree very well.
- First KLOE results from Radiative Return for the ρ region agree with CMD-2 (not τ).
- More data (and theoretical work) is needed (and expected) to clarify the situation.

II. $\Delta\alpha_{\rm QED}^{\rm had}$

- $\alpha_{\rm QED}$ the least well known parameter of (G_μ, M_Z and $\alpha(M_Z^2)$).
  Precision Electro-Weak fits affected! $\longrightarrow$ SM fit

- PDG2002: $\alpha^{-1} \equiv \alpha(0)^{-1} = 137.03599976(50)$,

$$\alpha(s)^{-1} = \Big(1 - \Delta\alpha_{\rm lep}(s) - \Delta\alpha_{\rm had}^{(5)}(s) - \Delta\alpha^{\rm top}(s)\Big)\,\alpha^{-1}$$

- $\Delta\alpha_{\rm lep}(M_Z^2) = 0.03149769$ (3 loop, $\rightarrow$ Steinhauser), $\Delta\alpha^{\rm top}(M_Z^2) = -0.000076$.

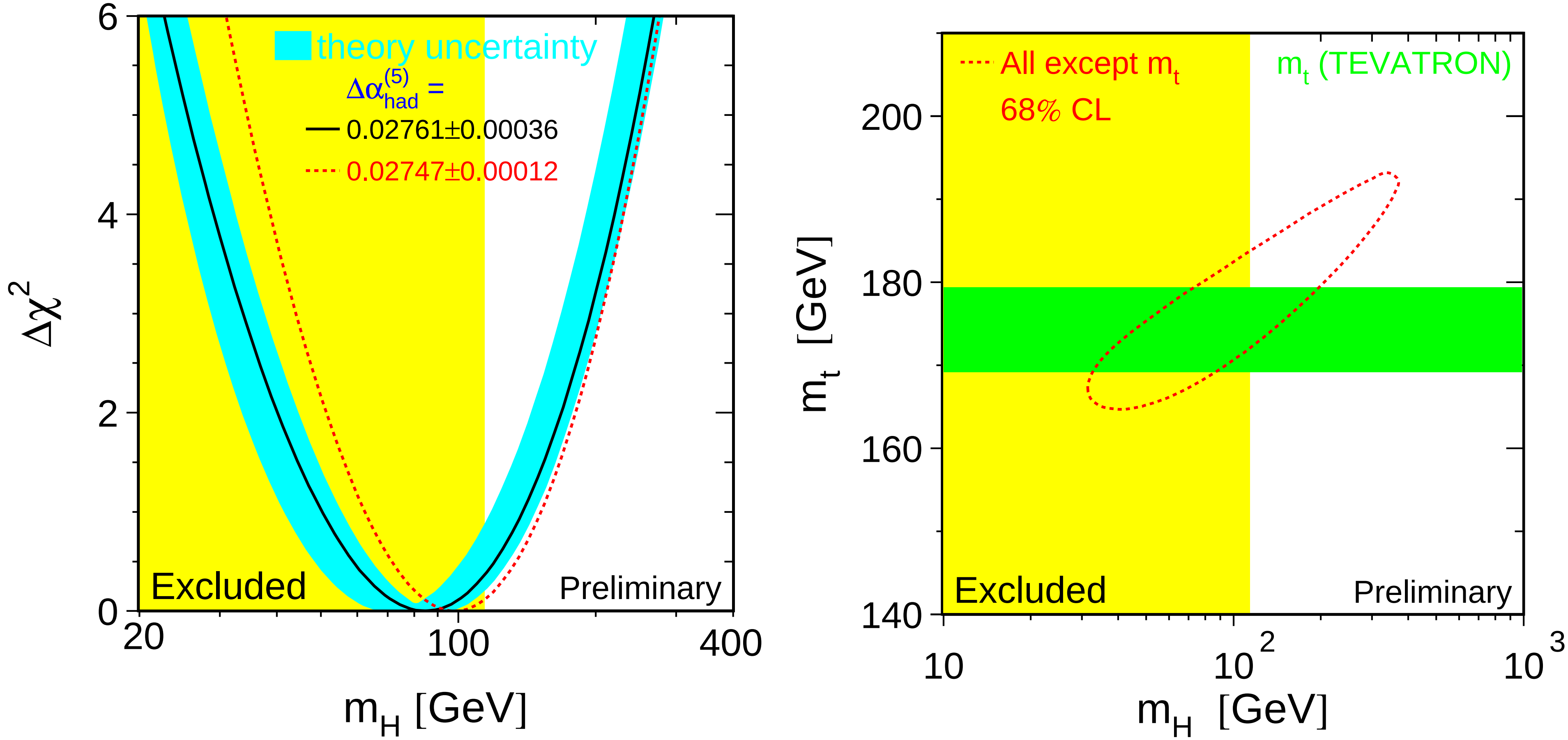
- Use of similar dispersion relation:

$$\boxed{\Delta\alpha_{\rm had}^{(5)} = -\frac{\alpha s}{3\pi}\, P\int_{s_{\rm th}}^{\infty} \frac{R(s')\, ds'}{s'(s'-s)}}$$

- HMNT find (preliminary, incl.):

$$\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.02769 \pm 0.00018\,, \quad \alpha(M_Z^2)^{-1} = 128.936 \pm 0.026\,.$$

Fitting the Higgs mass: (Plots from the LEP EWWG.)



$m_H = 88^{+50}_{-30}$ GeV, $m_H < 190$ GeV at 95% confidence.

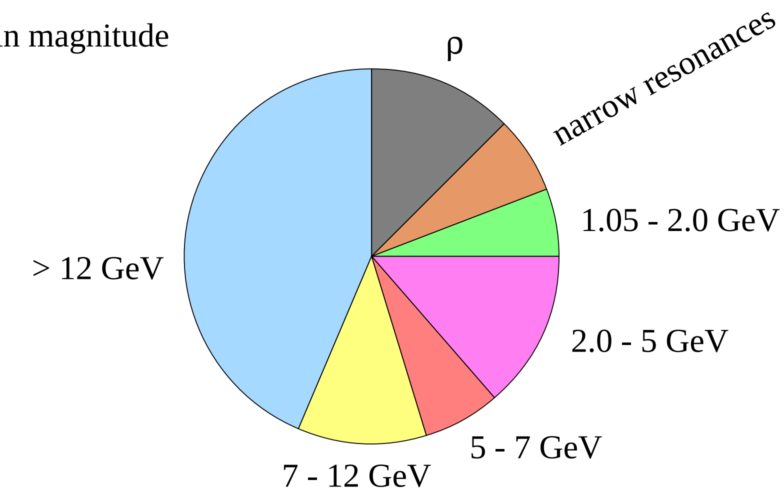
contributions at m_Z Burkhardt, Pietrzyk 2001

Contr. from the different energy regimes:

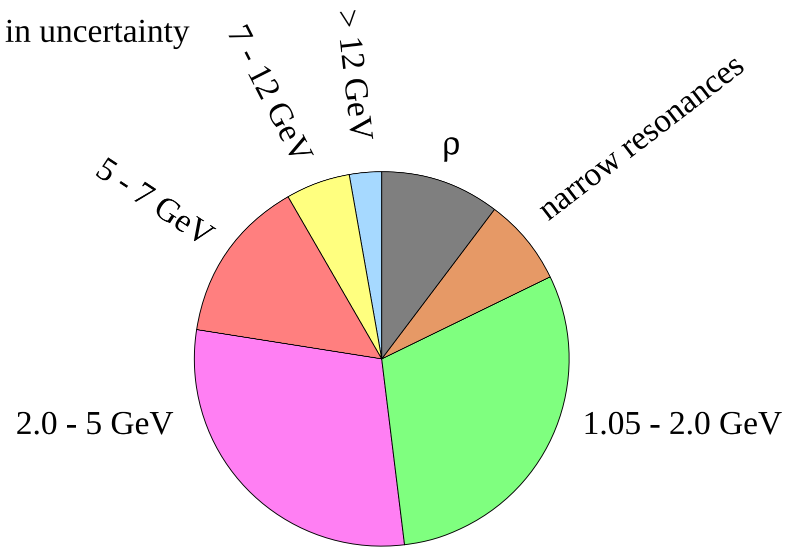
(HMNT prel.)

energy (GeV)	comments	$\Delta\alpha^{(5)}_{\rm had} \times 10^4$
$2m_\pi$—0.32	ch. pert. th.	0.04 ± 0.00
0.32—1.43	excl.	47.38 ± 0.34
1.43—2.00	incl.	10.94 ± 0.82
(1.43—2.00	excl.	12.26 ± 0.58)
2.00—11.09	incl.	82.48 ± 1.53
$J/\psi + \psi(2S)$	NW appr.	9.36 ± 0.62
$\Upsilon(1-6S)$	NW appr.	1.16 ± 0.04
11.09—∞	pert. QCD	125.49 ± 0.35
$\sum$	'incl.'	276.85 ± 1.92

in magnitude



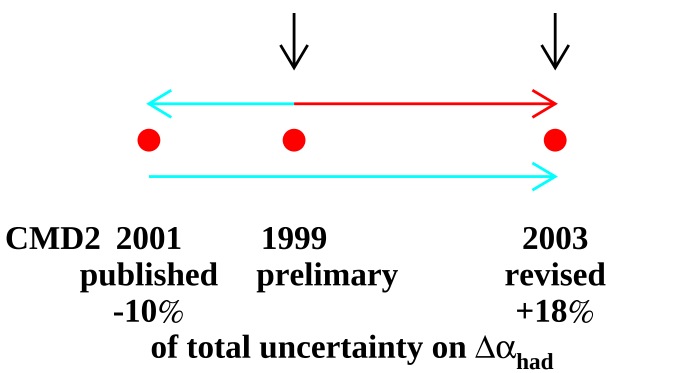
in uncertainty



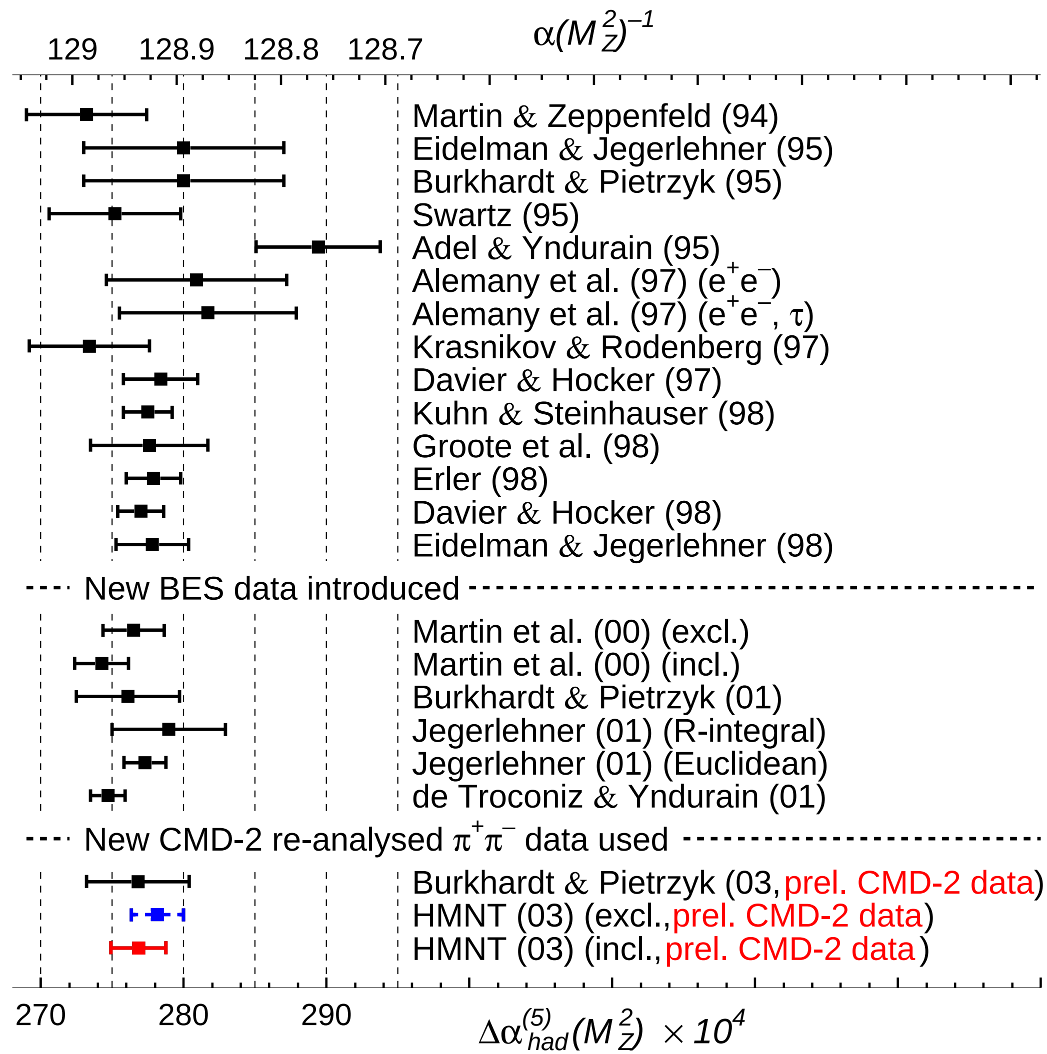
Influence of CMD-2 $\pi\pi$:

Burkhardt, Pietrzyk

	2001 published	2003 preliminary
$\Delta\alpha_{had}$	0.02761±0.00036	0.02768±0.00036



Comparison of different evaluations:



HMNT: Similar value but small error despite data-driven!

III. Future improvements / changes

- ⋆ Th + Exp: Further improved treatment of radiative corrections and correlations.
- ⋆ Th ? or Exp ?: Solving the τ puzzle.
- ⋆ Th + Exp (+ Lattice ?): improving the determination of the hadronic light-by-light contribution! Good ideas needed...
- • Expect more (precise) e^+e^- data from Novosibirsk (CMD-2 and SND), Beijing (BES II+III) and Cornell (CLEO(c)). (See talks on Friday.)
- ♢ Radiative Return analyses from KLOE, BaBar and BELLE will provide independent cross-section measurements over a wide energy region! (See next talks.)
- ♣ Waiting for the BNL analysis of the 3×10^9 data from μ^- (exp. error may decrease to $6 \cdot 10^{-10}$; tests CPT :–)
- ♡ Possibility of a new, even more precise g-2 experiment?!
- ▶ STAY TUNED!