Higher-Order Results in the Electroweak Theory

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Aachen, 07/2003

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1. Introduction

Electroweak precision measurements:

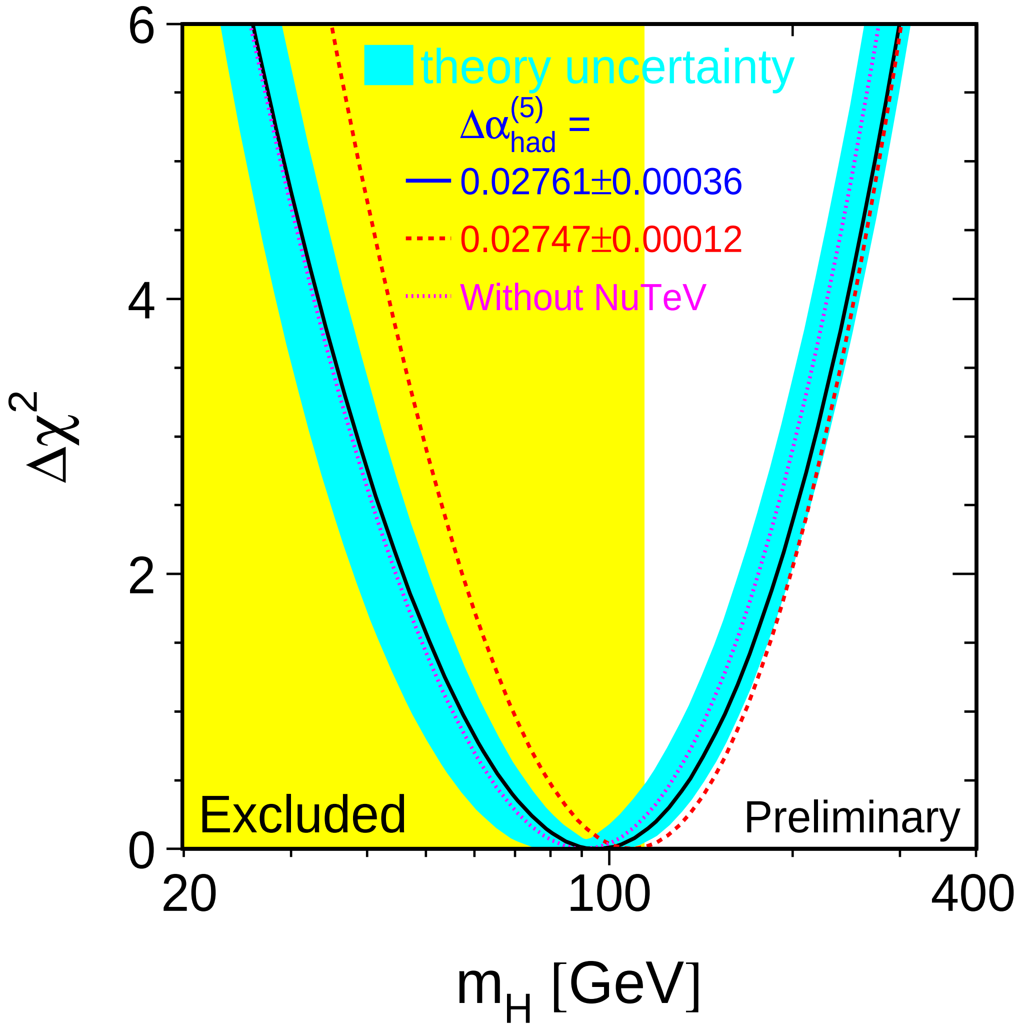
M_Z[GeV]	$=$	91.1875 ± 0.0021	0.002%
G_μ[GeV^{-2}]	$=$	$1.16637(1)\,10^{-5}$	0.0009%
m_t[GeV]	$=$	174.3 ± 5.1	2.9%
M_W[GeV]	$=$	80.426 ± 0.034	0.04%
$\sin^2\theta_{eff}^{lept}$	$=$	0.23150 ± 0.00016	0.07%
Γ_Z[GeV]	$=$	2.4952 ± 0.0023	0.09%
...			

Quantum effects of the theory:

Loop corrections to "pseudo-observables" M_W, $\sin^2\theta_{eff}$, Γ_Z, ...: $\sim \mathcal{O}(1\%)$

$\Rightarrow$ Indirect constraints on M_H, ... — effects of "new physics"?

Global fit to all data in the SM: [*LEPEWWG '03*]



Theoretical uncertainties:

– unknown higher-order corrections

$\Rightarrow$ "blue band"

– exp. error of input parameters:

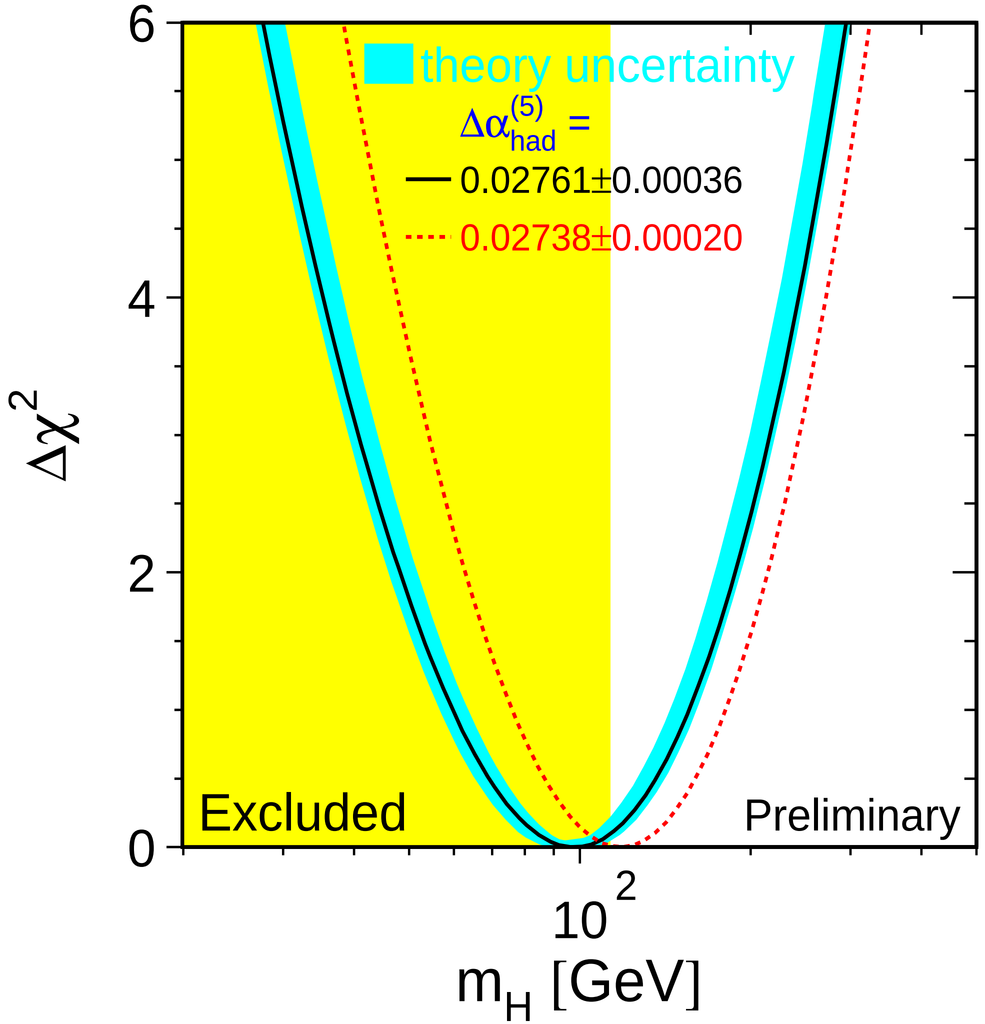
m_t, $\Delta\alpha_{had}$, ...

Overall fit probability (quality of the fit): 4.4% (27%) with (without) NuTeV

$$\Rightarrow M_H = 91^{+58}_{-37}\ \text{GeV}, \qquad M_H < 211\ \text{GeV, 95\% C.L.}$$

Uncertainties from unknown higher-order corrections:

Comparison: global fit results Winter '01 [*LEPEWWG '01*]



$\Rightarrow$ $M_H = 98^{+58}_{-38}$ GeV

$M_H < 212$ GeV, 95% C.L.

$\Rightarrow$ "Blue band" has widened

2. Present status of electroweak precision observables

Theoretical predictions for M_W, $\sin^2\theta_\mathrm{eff}$, ... in the SM:

Definition of Fermi constant G_μ via muon lifetime:

$$\tau_\mu^{-1} = \frac{G_\mu^2\, m_\mu^5}{192\pi^3}\, F\left(\frac{m_\mathrm{e}^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_\mathrm{W}^2}\right)(1+\Delta q)$$

Δq: QED corrections in Fermi Model [*R. Behrends, R. Finkelstein, A. Sirlin '56*] [*T. van Ritbergen, R. Stuart '99*] [*M. Steinhauser, T. Seidensticker '99*]

Comparison with SM prediction for muon decay:

$$\Rightarrow \quad M_\mathrm{W}^2\left(1 - \frac{M_\mathrm{W}^2}{M_\mathrm{Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1+\Delta r)\,,$$

$\Updownarrow$

loop corrections

$\Rightarrow$ Theoretical prediction for M_W in terms of M_Z, α, G_μ, $\Delta r(m_\mathrm{t}, M_\mathrm{H}, \ldots)$

Effective couplings at the Z resonance:

$$\Rightarrow \quad \sin^2\theta_{\text{eff}} = \frac{1}{4}\left(1 - \frac{\text{Re}\, g_V}{\text{Re}\, g_A}\right) = \left(1 - \frac{M_W^2}{M_Z^2}\right)\kappa(M_W^2)$$

One-loop result for M_W in the SM:

[*A. Sirlin '80*], [*W. Marciano, A. Sirlin '80*]

$$\Delta r_{1-\text{loop}} = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{\text{rem}}(M_H)$$

$$\sim \log\frac{M_Z}{m_f} \qquad \sim m_t^2$$

$$\sim 6\% \qquad \sim 3.3\% \qquad \sim 1\%$$

Leading contributions to M_W, $\sin^2\theta_{\text{eff}}$, ... from mass splitting between isospin doublet fields enter via

$$\Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}, \quad \Delta M_W \approx \frac{M_W}{2}\frac{c_W^2}{c_W^2 - s_W^2}\Delta\rho, \quad \Delta\sin^2\theta_{\text{eff}} \approx -\frac{c_W^2 s_W^2}{c_W^2 - s_W^2}\Delta\rho$$

Two-loop QCD corrections:

[*A. Djouadi, C. Verzegnassi '87*], [*A. Djouadi '89*], [*B. Kniehl '90*], [*F. Halzen, B. Kniehl '91*]

Electroweak two-loop corrections:

<u>$\sin^2\theta_{\text{eff}}$:</u>

Expansion for asymptotically large m_{t}:

Leading $m_{\text{t}}^4/M_{\text{W}}^4$ terms (enter via $\Delta\rho$)

[*J. van der Bij, F. Hoogeveen '87*], [*R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Vicere '92*], [*J. Fleischer, O.V. Tarasov, F. Jegerlehner '93*]

Next-to-leading $m_{\text{t}}^2/M_{\text{W}}^2$ terms [*G. Degrassi, P. Gambino, A. Sirlin '97*]

Expansions: $M_{\text{W}}, M_{\text{Z}}, M_{\text{H}} \ll m_{\text{t}}$, $M_{\text{W}}, M_{\text{Z}} \ll m_{\text{t}}, M_{\text{H}}$ + interpolation

$\rightarrow$ Same order of magnitude as $m_{\text{t}}^4/M_{\text{W}}^4$ terms, same sign

Expansion for asymptotically large M_{H}:

Leading $M_{\text{H}}^2/M_{\text{W}}^2$ terms (enter via $\Delta\rho$)

[*J. van der Bij, M. Veltman '84*], [*J. van der Bij '84*]

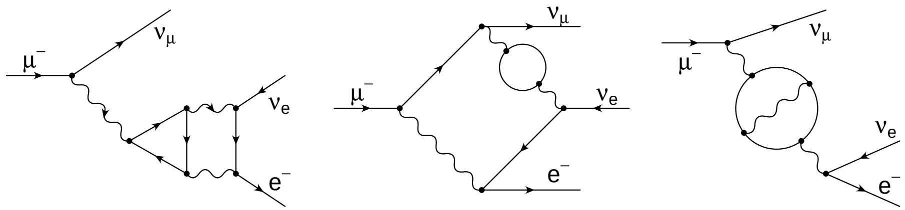
M_W:

Complete electroweak two-loop result:

Fermionic two-loop contributions:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

[*M. Awramik, M. Czakon '03*]



$\Rightarrow$ full dependence on m_t, complete light fermion contributions

Method:

Extraction of QED contributions in the Fermi model

Tensor integral reduction for general two-loop 2-point functions

[*G. W. '92*], [*G. W., R. Scharf, M. Böhm '94*]

General R_ξ gauge, careful treatment of γ_5 in dimensional regularisation
[*G. 't Hooft, M. Veltman '72*], [*P. Breitenlohner, D. Maison '77*]

Two-loop on-shell renormalization (physical parameters)

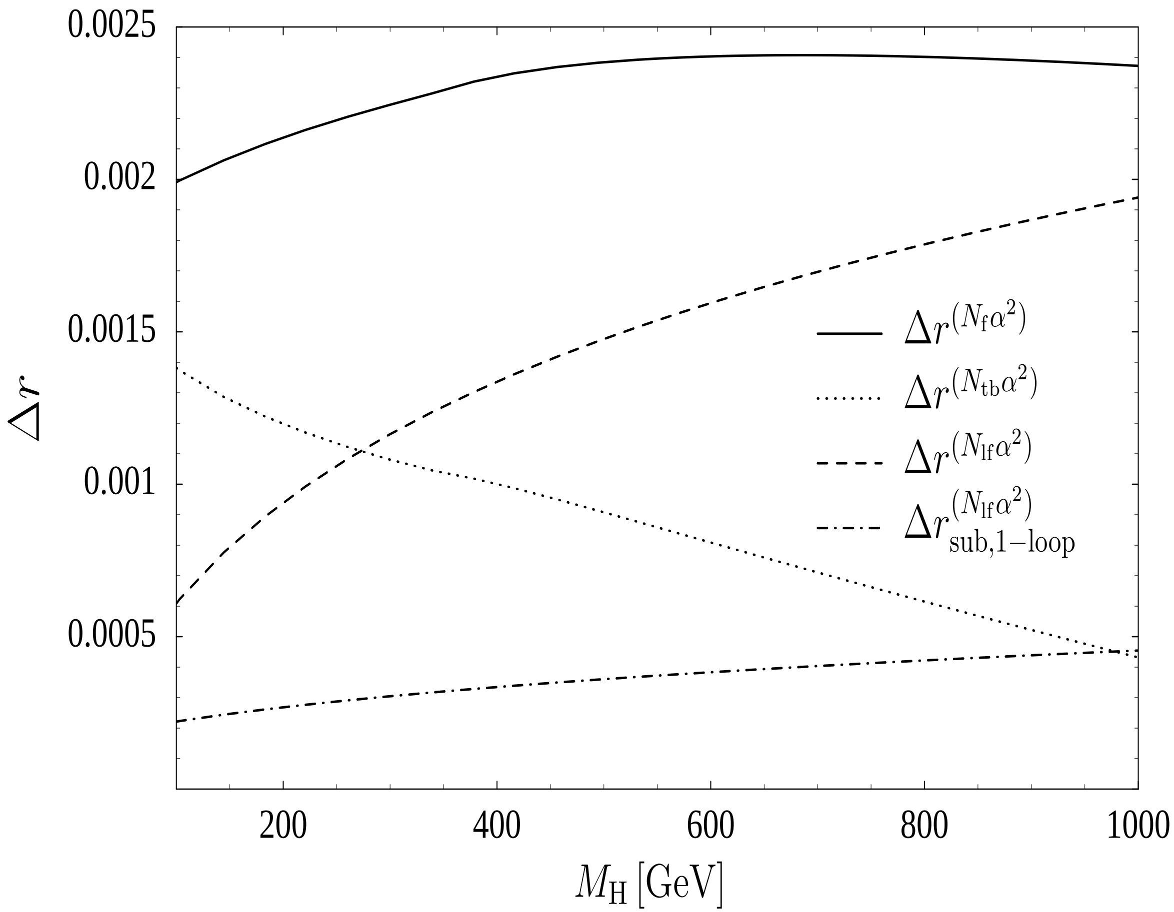
renormalization in gauge-fixing and ghost sector

mass renormalization of unstable particles
$\Rightarrow$ gauge-invariant definition according to real part of complex pole

Numerical evaluation via one-dimensional integral representations with elementary functions, results for arbitrary mass values
[*S. Bauberger, F. Berends, M. Böhm, M. Buza '95*], [*S. Bauberger, M. Böhm '95*]

Comparison of contributions from top/bottom doublet and from light fermions:

[*A. Freitas, W. Hollik, W. Walter, G.W. '02*]

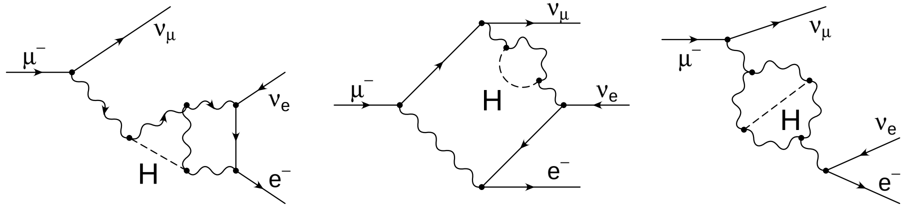


$$\Delta r^{(N_{\rm lf}\alpha^2)}_{\rm sub,1-loop} = \Delta r^{(N_{\rm lf}\alpha^2)} - 2\Delta\alpha\Delta r^{(\alpha)}_{\rm bos}$$

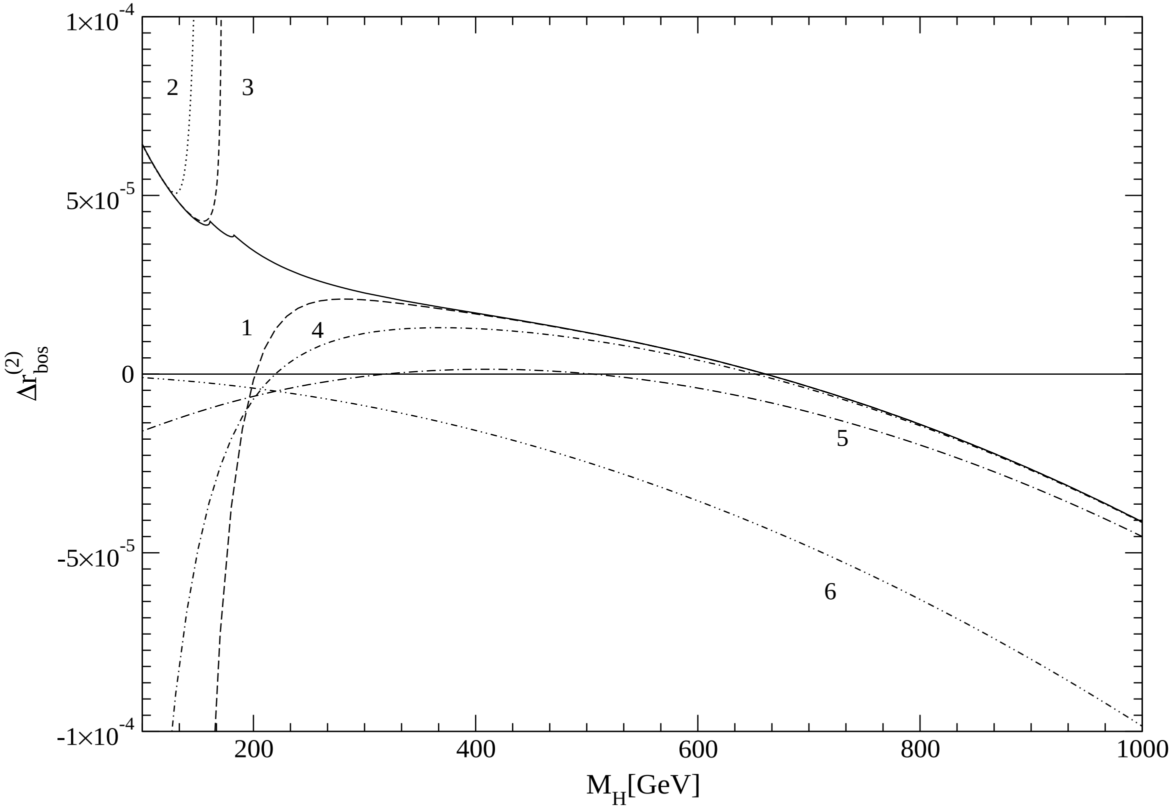
$\Rightarrow$ non-leading t–b and genuine 2-loop light fermion contributions lead to shift in $M_{\rm W}$ of about 5 MeV

Purely bosonic two-loop contributions:

[*M. Awramik, M. Czakon '02*] [*A. Onishchenko, O. Veretin '02*]



Similar methods as for fermionic contributions, numerical integration & expansion in $(M_H^2 - M_Z^2)$, $(M_Z^2 - M_W^2)$ and for $M_H \gg M_Z$



$\Rightarrow \Delta M_W \lesssim \pm 1$ MeV

Three-loop QCD corrections to M_W and $\sin^2\theta_\mathrm{eff}$:

[*L. Avdeev, J. Fleischer, S.M. Mikhailov, O. Tarasov '94*]

[*K. Chetyrkin, J. Kühn, M. Steinhauser '95*]

Leading EW corrections beyond 2-loop:

Pure fermion-loop contributions up to 4-loop order

[*A. Stremplat '98*], [*G.W. '98*]

Leading 3-loop corrections to $\Delta\rho$:

$\mathcal{O}(G_\mu^3 m_\mathrm{t}^6)$, $\mathcal{O}(G_\mu^2 \alpha_\mathrm{s} m_\mathrm{t}^4)$ terms, limit: $M_\mathrm{H} = 0$

[*J. van der Bij, K. Chetyrkin, M. Faisst, G. Jikia, T. Seidensticker '01*]

$\mathcal{O}(G_\mu^3 m_\mathrm{t}^6)$, $\mathcal{O}(G_\mu^2 \alpha_\mathrm{s} m_\mathrm{t}^4)$ terms, arbitrary M_H

[*M. Faisst, J. Kühn, T. Seidensticker, O. Veretin '03*]

Method:

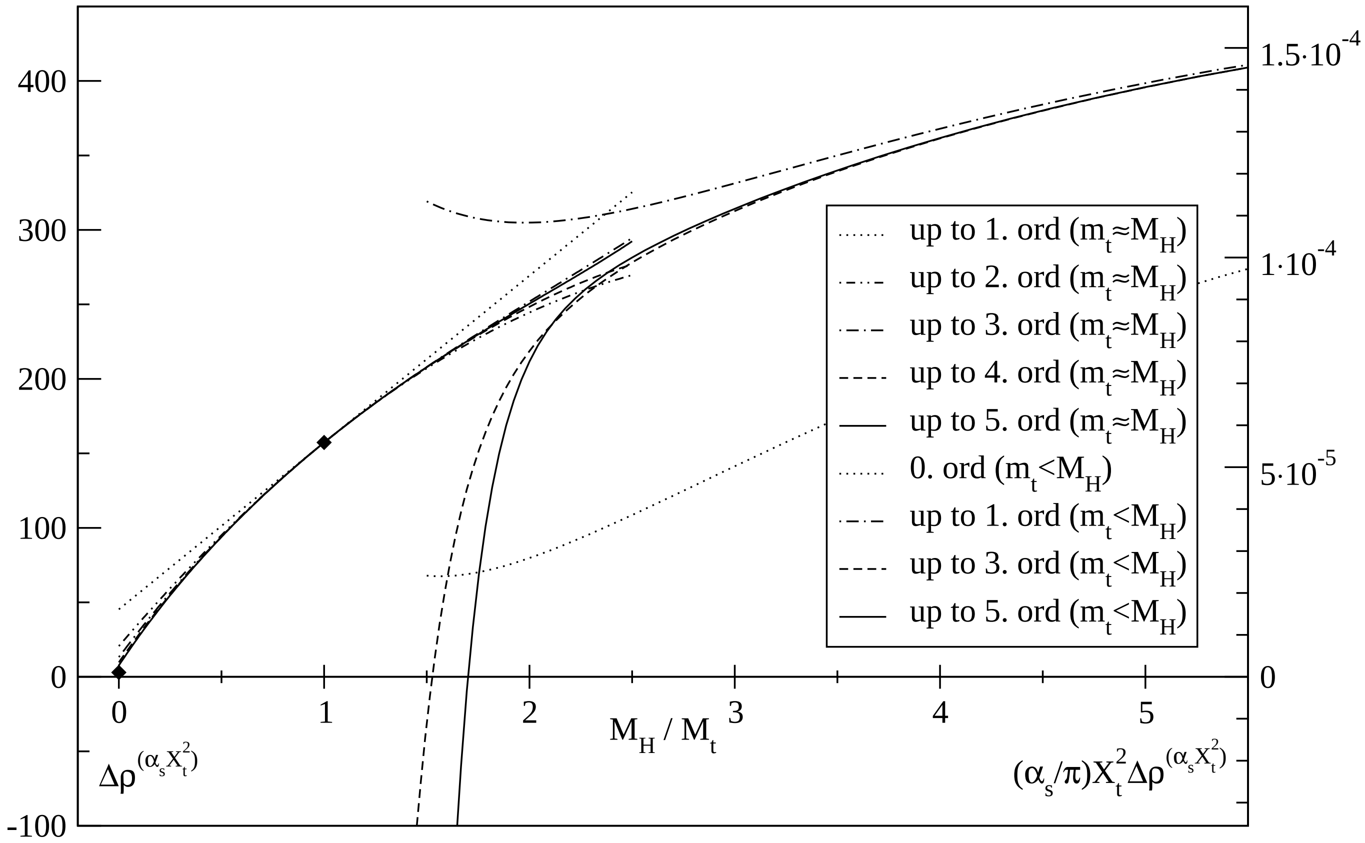
Expansions around $M_\mathrm{H} = m_\mathrm{t}$ and for $M_\mathrm{H} \gg m_\mathrm{t}$

$\Rightarrow$ three-loop vacuum integrals with one mass scale

Transition between $\overline{\text{MS}}$ and on-shell value of top-quark mass

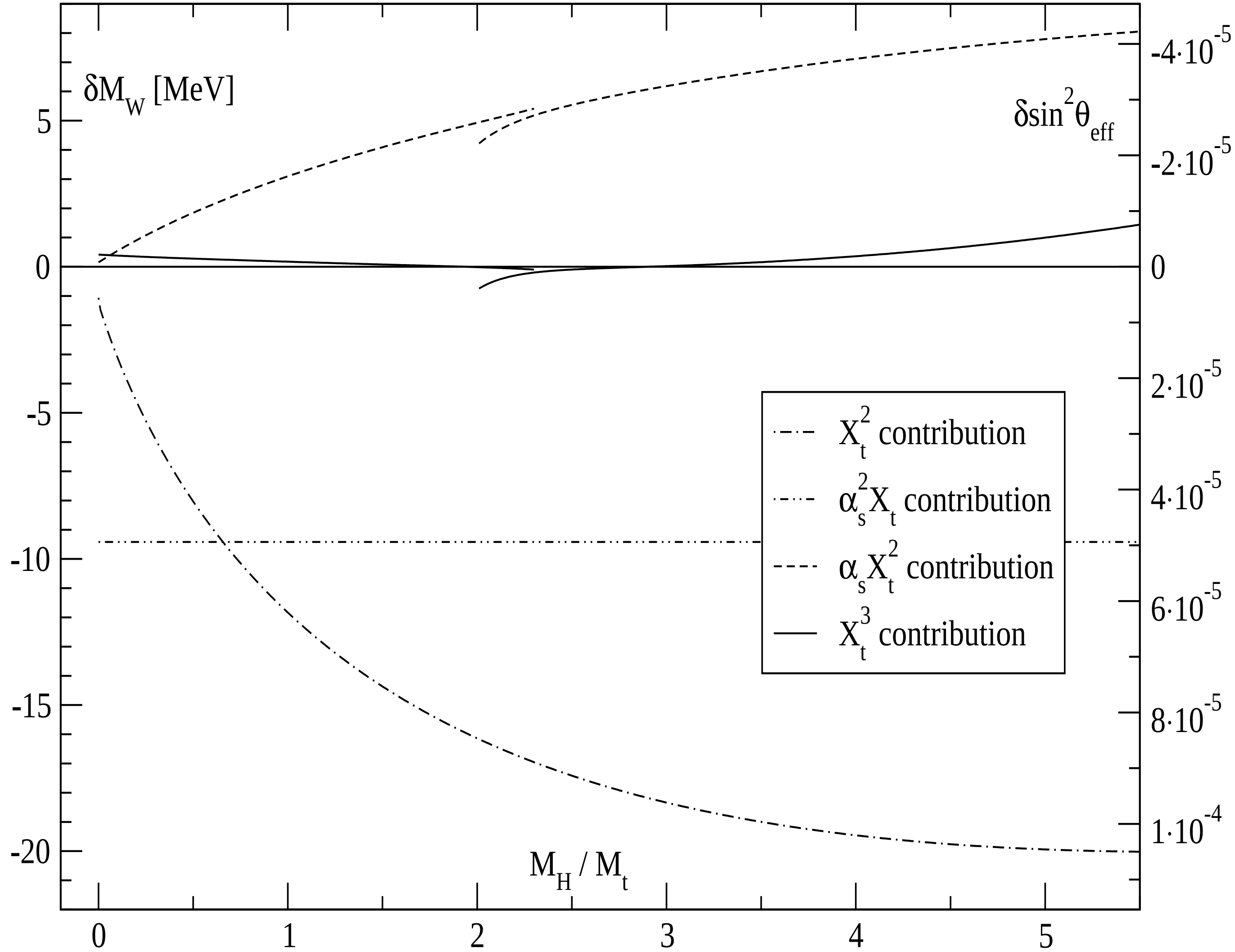
$\Rightarrow$ two-loop top-quark self-energy on-shell, evaluated with expansions around $M_{\text{H}} = m_{\text{t}}$ and for $M_{\text{H}} \gg m_{\text{t}}$

$\mathcal{O}(G_\mu^2 \alpha_{\text{s}} m_{\text{t}}^4)$ contributions to $\Delta\rho$:



Shifts in M_W, $\sin^2\theta_{\rm eff}$ from $\mathcal{O}(G_\mu^3 m_t^6)$, $\mathcal{O}(G_\mu^2 \alpha_s m_t^4)$ contributions to $\Delta\rho$

[*M. Faisst, J. Kühn, T. Seidensticker, O. Veretin '03*]



$\Rightarrow \Delta M_W \lesssim 5$ MeV, $\Delta \sin^2\theta_{\rm eff} \lesssim -3 \times 10^{-5}$

Simple parameterization of full result for M_W (contains all known corrections): [*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

$$\begin{aligned} M_\mathrm{W} \;=\; & M_\mathrm{W}^0 - c_1\,\mathrm{dH} - c_2\,\mathrm{dH}^2 + c_3\,\mathrm{dH}^4 + c_4\,\mathrm{dh} \\ & - c_5\,\mathrm{d}\alpha + c_6\,\mathrm{dt} - c_7\,\mathrm{dt}^2 - c_8\,\mathrm{dH}\,\mathrm{dt} \\ & + c_9\,\mathrm{dh}\,\mathrm{dt} - c_{10}\,\mathrm{d}\alpha_\mathrm{s} + c_{11}\,\mathrm{dz} \end{aligned}$$

where

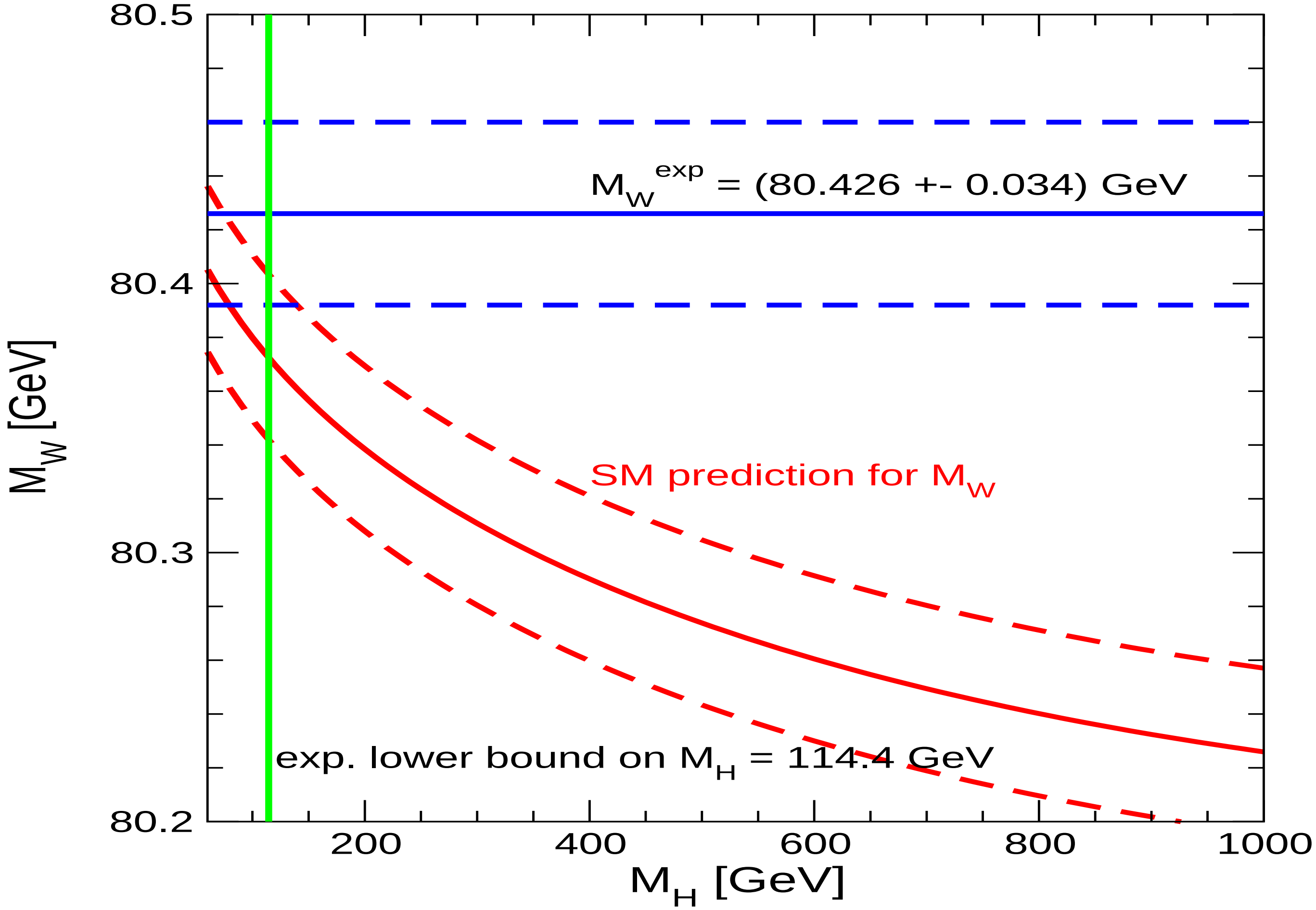
$$\mathrm{dH} = \ln\left(\frac{M_\mathrm{H}}{100\ \mathrm{GeV}}\right), \quad \mathrm{dh} = \left(\frac{M_\mathrm{H}}{100\ \mathrm{GeV}}\right)^2 - 1, \quad \mathrm{dt} = \left(\frac{m_\mathrm{t}}{174.3\ \mathrm{GeV}}\right)^2 - 1$$

$$\mathrm{d}\alpha = \frac{\Delta\alpha}{0.05907} - 1, \quad \mathrm{d}\alpha_\mathrm{s} = \frac{\alpha_\mathrm{s}(M_\mathrm{Z})}{0.119} - 1, \quad \mathrm{dz} = M_\mathrm{Z} - 91.1875\ \mathrm{GeV}$$

⇒ approximates full result within 0.5 MeV for 10 GeV $\leq M_\mathrm{H} \leq$ 1 TeV, 2σ variations

SM prediction for M_W vs. experimental result:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]



$\Rightarrow$ Light Higgs preferred

$M_W^{\text{exp}} = 80.426 \pm 0.034$ GeV, $m_t^{\text{exp}} = 174.3 \pm 5.1$ GeV,

$M_H > 114.4$ GeV, 95% C.L.

$\Rightarrow 1\sigma$ regions hardly overlap

3. Remaining theoretical uncertainties

Uncertainties from unknown higher-order corrections, different methods:

"Traditional Blue Band method":
[*D. Bardin, M. Grünewald, G. Passarino '99*]

Size of unknown corrections estimated by varying different options in *ZFITTER* [*D. Bardin et al.*], *TOPAZ0* [*G. Passarino*]

Different renormalization schemes, different resummations of non-leading terms
$\Rightarrow$ differ by higher-order corrections

difficult to quantify, genuine effects of irreducible higher-order corrections not accessible

Error estimate for blue band plot up to Moriond '01: [*LEPEWWG '01*]

$$\Delta M_{\mathrm{W}}^{\mathrm{theo}} \approx \pm 3 \text{ MeV}, \quad \Delta \sin^2\theta_{\mathrm{eff}}^{\mathrm{theo}} \approx \pm 3 \times 10^{-5}$$

More conservative estimate for $\sin^2\theta_{\text{eff}}$ (since summer '01) [*LEPEWWG '01*]

Effect of non-leading fermionic and purely bosonic two-loop corrections on $\sin^2\theta_{\text{eff}}$ not yet known

$\Rightarrow$ use corresponding shift in M_{W} to estimate uncertainty of $\sin^2\theta_{\text{eff}}$ according to $\sin^2\theta_{\text{eff}} = \left(1 - \frac{M_{\text{W}}^2}{M_{\text{Z}}^2}\right)\kappa(M_{\text{W}}^2)$

$\Rightarrow$ upward shift in $\sin^2\theta_{\text{eff}}$ of $\Delta\sin^2\theta_{\text{eff}}^{\text{theo}} \approx 8\times 10^{-5}$ for $M_{\text{H}} = 100$ GeV

Estimate of QCD corrections:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

– on-shell vs. $\overline{\text{MS}}$ value of m_{t}

– estimate $\Delta r^{(\alpha^2\alpha_{\text{s}})}/\Delta r^{(\alpha^2)}$ from known $\Delta r^{(\alpha\alpha_{\text{s}})}/\Delta r^{(\alpha)}$

$\Rightarrow$ $\mathcal{O}(\alpha^2\alpha_{\text{s}})$: $\Delta M_{\text{W}}^{\text{theo}} \approx 4$ MeV, $\mathcal{O}(\alpha\alpha_{\text{s}}^3)$: $\Delta M_{\text{W}}^{\text{theo}} \approx 1$ MeV

Transition from fixed-width definition to running width definition of M_{W}:

[*A. Freitas, W. Hollik, W. Walter, G.W. '00, '02*]

$\Delta M_{\text{W}}^{\text{theo}} \approx 1$ MeV

Estimate according to possible enhancement factors multiplied by coefficients of $\mathcal{O}(1)$

[*U. Baur, R. Clare, J. Erler, S. Heinemeyer, D. Wackeroth, G. W., D. Wood '01*]

order	sector	estimate	M_W[MeV]	$\sin^2\theta_{eff}$ (10^5)
α^2	fermionic	$N(\alpha/4\pi\hat{s}^2)^2$	complete	4.1
α^2	bosonic	$(\alpha/\pi\hat{s}^2)^2$	2.1	4.1
$\alpha\alpha_s^2$	t–b doublet	$N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	complete	complete
$\alpha\alpha_s^2$	light doubl.	$2\ N_C C_F C_A \alpha\alpha_s^2/4\pi^3\hat{s}^2$	complete	complete
$\alpha^3 m_t^6$	heavy top	$5.3\ N_C^2(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	4.1	2.3
$\alpha^3 m_t^6$	heavy top	$3.3\ N_C(\alpha m_t^2/4\pi\hat{s}^2 M_W^2)^3$	0.9	0.5
$\alpha^2\alpha_s m_t^4$	heavy top	$3.9\ N_C C_F \alpha^2\alpha_s m_t^4/16\pi^3\hat{s}^4 M_W^4$	4.5	2.5
$\alpha\alpha_s^3 m_t^2$	heavy top	$N_C C_F C_A^2 \alpha\alpha_s^3 m_t^2/4\pi^4\hat{s}^2 M_W^2$	1.3	0.8
	total		7	7

$\Rightarrow$ good agreement for the corrections that meanwhile have been calculated

Estimate of remaining uncertainties from unknown higher orders in M_W and $\sin^2\theta_{\rm eff}$:

[*M. Awramik, M. Czakon, A. Freitas, G.W. '03*]

M_W: $\mathcal{O}(G_\mu^3 m_t^4 M_Z^2)$, $\mathcal{O}(G_\mu^2 \alpha_s m_t^2 M_Z^2)$, $\mathcal{O}(\alpha\alpha_s^3)$

$$\Delta M_W^{\rm theo} \approx 4 \text{ MeV}$$

$\sin^2\theta_{\rm eff}$: non-leading fermionic two-loop contributions, purely bosonic two-loop contributions, $\mathcal{O}(G_\mu^3 m_t^4 M_Z^2)$, $\mathcal{O}(G_\mu^2 \alpha_s m_t^2 M_Z^2)$, $\mathcal{O}(\alpha\alpha_s^3)$

$$\Delta \sin^2\theta_{\rm eff}^{\rm theo} \approx 6 \times 10^{-5}$$

Uncertainties from experimental errors of input parameters:

$$\Delta m_{\rm t} = \pm 5.1 \text{ GeV} \;\Rightarrow\; \Delta M_{\rm W} \approx \pm 31 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff} \approx \pm 16 \times 10^{-5}$$

$$\Delta(\Delta\alpha_{\rm had}) = \pm 3.6 \times 10^{-4} \;\Rightarrow\; \Delta M_{\rm W} \approx \pm 6.5 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff} \approx \pm 13 \times 10^{-5}$$

$$\Delta\alpha_{\rm s} = \pm 0.002 \;\Rightarrow\; \Delta M_{\rm W} \approx \pm 1.5 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff} \approx \pm 2.5 \times 10^{-5}$$

$$\Delta M_{\rm Z} = \pm 2.1 \text{ MeV} \;\Rightarrow\; \Delta M_{\rm W} \approx \pm 2.5 \text{ MeV}, \quad \Delta \sin^2\theta_{\rm eff} \approx \pm 1.4 \times 10^{-5}$$

Possible future improvement in $\Delta(\Delta\alpha_{\rm had})$: $\rightarrow \pm 0.5 \times 10^{-4}$ [*F. Jegerlehner '01*]

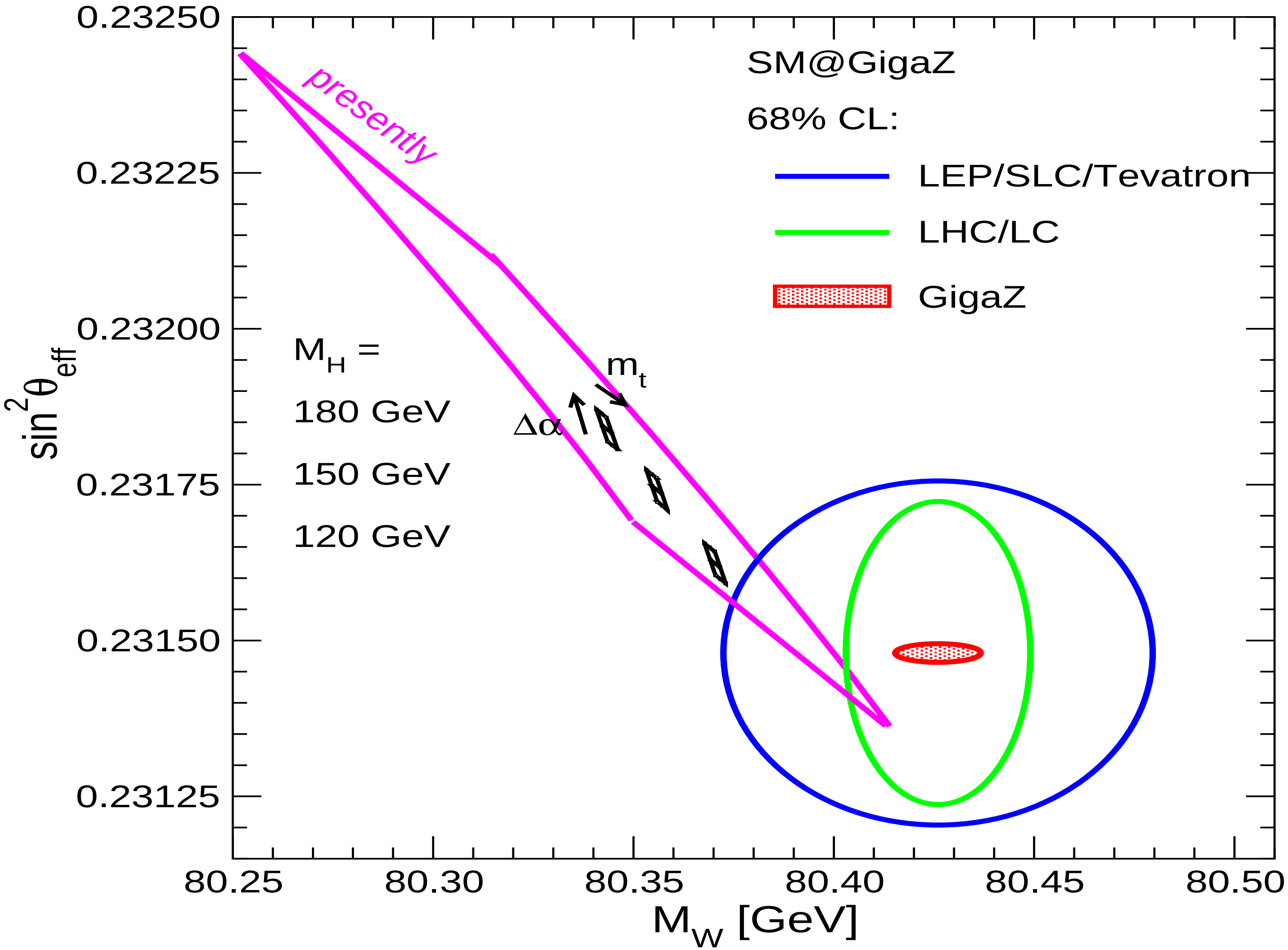
Prospects for experimental accuracies at the next generation of colliders:

$$\Delta m_{\rm t}^{\rm exp} / \text{ GeV}: \quad 5.1 \xrightarrow{\text{Tev/LHC}} 1.5 \xrightarrow{\text{LC}} 0.1$$

$$\Delta M_{\rm W}^{\rm exp} / \text{ MeV}: \quad 34 \xrightarrow{\text{Tev/LHC}} 15 \xrightarrow{\text{LC}} 7$$

$$\Delta \sin^2\theta_{\rm eff}^{\rm exp} / 10^{-5}: \quad 16 \xrightarrow{\text{Tev/LHC}} 16 \xrightarrow{\text{LC}} 1$$

SM prediction for M_W and $\sin^2\theta_{eff}$ vs. current experimental result (LEP2/Tevatron) and prospective accuracies at the LHC and a LC with low-energy option (GigaZ): [*J. Erler, S. Heinemeyer, W. Hollik, G.W., P. Zerwas '00*]



⇒ Sensitive test of the electroweak theory

4. Conclusions

- Higher-order results for M_{W}, $\sin^2\theta_{\mathrm{eff}}$ in the SM:

  M_{W}: complete two-loop result, leading QCD and ew three-loop corr.

  $\sin^2\theta_{\mathrm{eff}}$: non-leading two-loop contributions still missing

- Estimate of remaining theoretical uncertainties from unknown higher-order corrections:

  $\Delta M_{\mathrm{W}}^{\mathrm{theo}} \approx 4$ MeV, $\Delta\sin^2\theta_{\mathrm{eff}}^{\mathrm{theo}} \approx 6 \times 10^{-5}$

- Effect of parametric uncertainty of m_{t}:

  present: $\Delta M_{\mathrm{W}} \approx \pm 31$ MeV, $\Delta\sin^2\theta_{\mathrm{eff}} \approx \pm 16 \times 10^{-5}$

  LC: $\Delta M_{\mathrm{W}} \approx \pm 1$ MeV, $\Delta\sin^2\theta_{\mathrm{eff}} \approx \pm 0.3 \times 10^{-5}$

- At next generation of colliders:

  Improved accuracy of precision observables M_{W}, $\sin^2\theta_{\mathrm{eff}}$, m_{h}, ... and input parameters m_{t}, $m_{\tilde{\mathrm{t}}}$, ...

  $\Rightarrow$ Very sensitive test of electroweak theory