Theoretical status of CP violation in Kaon decays $[\epsilon'/\epsilon]$

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* Recent developments in analytic approaches [$\langle Q_8 \rangle_2$, $\langle Q_6 \rangle_0$, IB]
* Comparison with available lattice results

EPS-HEP2003, Aachen, July 17-23 2003

Parallel session 12: CP Violation

CP Violation in $K^0 \to \pi\pi$

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \simeq \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \simeq \epsilon - 2\,\epsilon'$$

CP-viol. in $K - \bar{K}$ Mixing:

$$|\epsilon| = (2.282 \pm 0.017) \times 10^{-3}$$
$$\Phi_\epsilon = (43 \pm 0.5)^o$$

CP-viol. in $K \to \pi\pi$ Amplitudes:

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2\right) = \underbrace{(16.6 \pm 1.6) \times 10^{-4}}_{\text{2002 WA}}$$

$K \to \pi\pi$ isospin amplitudes

$$|\pi\pi\rangle : \qquad J = 0, \quad CP = +, \quad I = 0, 2 \quad \text{(Bose)}$$

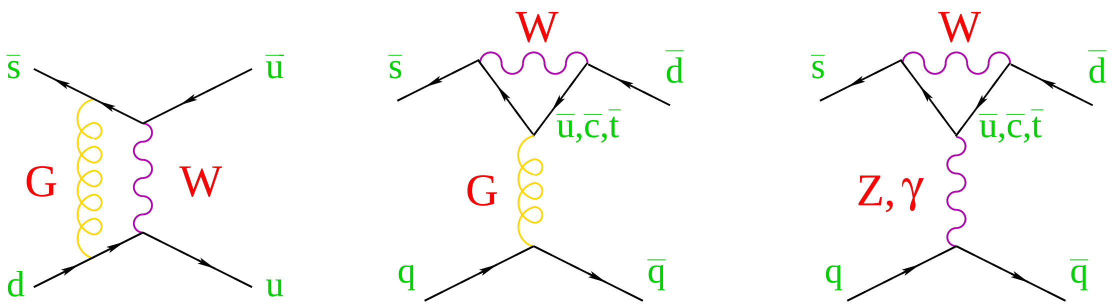
$$\begin{aligned}
A[K^0 \to \pi^+\pi^-] &= A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2} \\
A[K^0 \to \pi^0\pi^0] &= A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2} \\
A[K^\pm \to \pi^\pm\pi^0] &= \frac{3}{2} A_2{}^+\, e^{i\chi_2^+}
\end{aligned}$$

$$\chi_0 - \chi_2 \sim 50^o \quad \text{(strong phases)}$$

$$\omega_+ = \frac{\mathrm{Re} A_2^+}{\mathrm{Re} A_0} \sim \frac{1}{22.1} \quad (\Delta I = \frac{1}{2} \text{ rule})$$

$$\frac{\epsilon'}{\epsilon} = \frac{-i e^{i(\chi_2 - \chi_0 - \Phi_\epsilon)}}{\sqrt{2}|\epsilon|} \frac{\mathrm{Re} A_2}{\mathrm{Re} A_0} \left[\frac{\mathrm{Im} A_0}{\mathrm{Re} A_0} - \frac{\mathrm{Im} A_2}{\mathrm{Re} A_2} \right]$$

$|\Delta S| = 1$ Effective Hamiltonian



$$\mathcal{H}_{\rm eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu)\, Q_i(\mu)$$

$$C_i(\mu) = z_i(\mu) - y_i(\mu)\, \frac{V_{td}V_{ts}^*}{V_{ud}V_{us}^*}$$

$$Q_1 = (\bar{s}_\alpha u_\beta)_{\rm V-A}\ (\bar{u}_\beta d_\alpha)_{\rm V-A}$$

$$Q_2 = (\bar{s}u)_{\rm V-A}\ (\bar{u}d)_{\rm V-A}$$

$$Q_{3,5} = (\bar{s}d)_{\rm V-A} \sum_q (\bar{q}q)_{\rm V\mp A}$$

$$Q_{4,6} = (\bar{s}_\alpha d_\beta)_{\rm V-A} \sum_q (\bar{q}_\beta q_\alpha)_{\rm V\mp A}$$

$$Q_{7,9} = \frac{3}{2}(\bar{s}d)_{\rm V-A} \sum_q e_q (\bar{q}q)_{\rm V\pm A}$$

$$Q_{8,10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_{\rm V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{\rm V\pm A}$$

* $C_i(\mu) \sim \mathcal{O}(\alpha_S^n t^n, \alpha_S^{(n+1)} t^n)$
  $t = \log(M_Z/\mu)$ [Munich, Rome]

* $\langle \pi\pi | Q_i(\mu) | K \rangle \ \leftrightarrow$ Non-perturbative physics
  Need to keep track of scale (μ) dependence and ren. scheme

* Physics does not depend on μ

ϵ'/ϵ in the SM: basic ingredients

$$\left|\frac{\epsilon'}{\epsilon}\right| = \mathrm{Im}(V_{td}V_{ts}^*) \cdot \left[P^{(1/2)} - P^{(3/2)}\right]$$

$$P^{(1/2)} = r \sum_i y_i(\mu) \frac{\mathrm{Disp}[\langle Q_i(\mu)\rangle_0]}{\cos\chi_0}(1-\Omega_{\mathrm{IB}}) \quad \sim r\, y_6(\mu) \frac{\mathrm{Disp}[\langle Q_6(\mu)\rangle_0]}{\cos\chi_0}(1-\Omega_{\mathrm{IB}})$$

$$P^{(3/2)} = \frac{r}{\omega} \sum_i y_i(\mu) \frac{\mathrm{Disp}[\langle Q_i(\mu)\rangle_2]}{\cos\chi_2} \quad \sim \frac{r}{\omega} y_8(\mu) \frac{\mathrm{Disp}[\langle Q_8(\mu)\rangle_2]}{\cos\chi_2}$$

Phenomenology input **

$$\mathrm{Im}(V_{td}V_{ts}^*) = (1.31 \pm 0.1) \times 10^{-4}$$

$$r = \frac{G_F\,\omega}{2|\epsilon|\mathrm{Re}A_0}$$

$$\omega = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}$$

Theory input

1. $y_i(\mu) \leftrightarrow$ at NLL [Munich, Roma]
2. $\langle Q_6(\mu)\rangle_0, \quad \langle Q_8(\mu)\rangle_2$
3. $\Omega_{\mathrm{IB}} \leftrightarrow$ isospin-breaking effects

Hadronic matrix elements $\langle Q_i(\mu)\rangle_I$

Methods on the Market

- ◇ Lattice QCD (see talk by L. Giusti): eventually will be the most reliable
- ◇ Analytic approaches: $1/N_c$, quark models, disp. rels, and combinations

[Some of them suffer from serious shortcomings]

Organizing Principles

1. Chiral Symmetry and ChPT: $\langle Q_i(\mu)\rangle_I \sim (p/\Lambda_\chi)^{2n}, n = 0, 1, \ldots$
2. $1/N_c$ expansion: $N_c \equiv \infty \leftrightarrow$ factorization of Q_i. At LO in ChPT:

$$\langle Q_6(\mu)\rangle_0^\infty = -4\sqrt{2}\,(F_K - F_\pi)\left(\frac{M_K^2}{m_s(\mu) + m_d(\mu)}\right)^2$$

$$\langle Q_8(\mu)\rangle_2^\infty = 2\,F_\pi\left(\frac{M_K^2}{m_s(\mu) + m_d(\mu)}\right)^2$$

Deviations from factorization historically denoted by $B_i^I(\mu) = \frac{\langle Q_i(\mu)\rangle_I}{\langle Q_i(\mu)\rangle_I^\infty}$

Recent progress in calculating $\langle Q_i(\mu)\rangle_I$ (aside from lattice)

◇ $Q_{7,8}$ at $O(p^0)$ (chiral limit) beyond factorization.
Correct matching in μ and scheme dependence (at NLL level).

V. C., J. F. Donoghue, E. Golowich, K. Maltman, Phys. Lett. **B 555** (2003) 71, **B 522** (2001) 245 (CDGM)

M. Knecht, S. Peris, E. de Rafael Phys. Lett. **B 508** (2001) 117 (KPR)

J. Bijnens, E. Gamiz, J. Prades, JHEP **0110** (2001) 009 (BGP)

◇ $Q_{6,4}$ at $O(p^2)$ (leading chiral order) including $O(\frac{n_f}{N_c})$ corrections.
Correct matching in μ (at LL level).

T. Hambye, S. Peris, E. de Rafael JHEP **0305** (2003) 27 (HPR)

◇ Going beyond leading chiral order: final state interaction (FSI) and more

E. Pallante, A. Pich, I. Scimemi Nucl. Phys. **B 617** (2001) 441 (PPS)

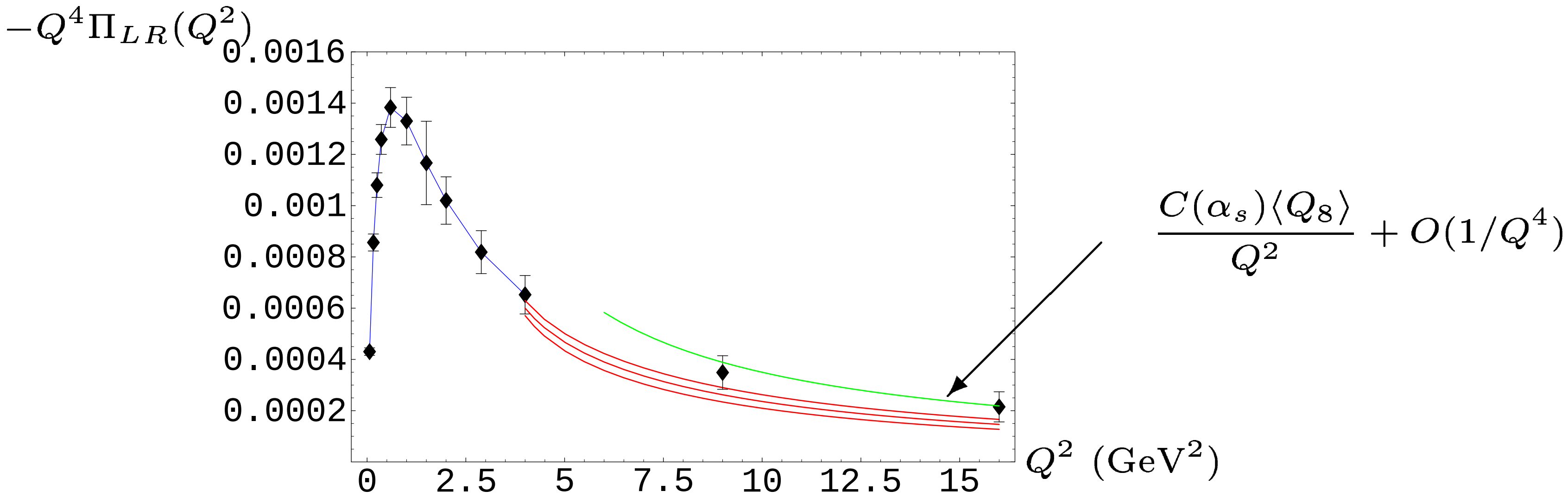
◇ Isospin-breaking ($\Omega_{\rm eff}$): first complete calculation at $O((m_u - m_d)p^2, e^2p^2)$.
NLO LECs determined in leading $1/N_c$, uncertainty studied.

V.C., G. Ecker, H. Neufeld, A. Pich hep-ph/0307030 (CENP)

Electroweak Penguins at leading chiral order (p^0) CDGM

$$Q_7(\mu) \to \langle \lambda U Q U^\dagger \rangle \left[\frac{3(d-1)\mu^{4-d}}{(4\pi)^{d/2}\Gamma(d/2)} \int_0^\infty dQ^2\, Q^d\, \Pi_{LR}(Q^2) \right]_{\overline{\rm MS}} \leftrightarrow \text{“}\overline{MS}\text{ area”}$$

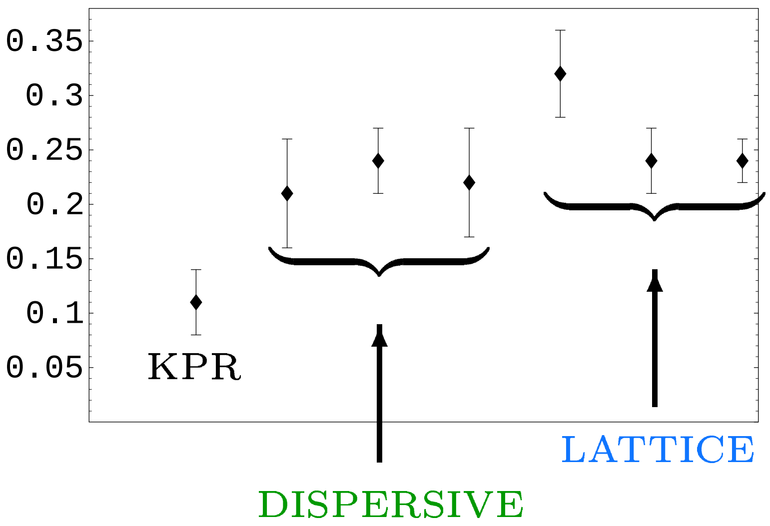
$$Q_8(\mu) \to \langle \lambda U Q U^\dagger \rangle \left[\frac{1}{C_{\overline{MS}}(\alpha_s(\mu))} \mu^6\, \Pi_{LR}(\mu^2) + O(1/\mu^2) \right] \leftrightarrow \text{“}\overline{MS}\text{ tail”}$$



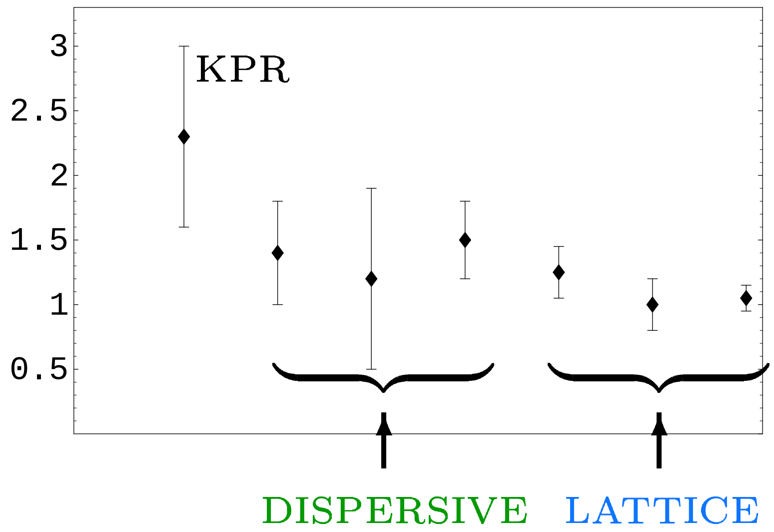
- ⋆ OPE calculation of $\Pi_{LR}(Q^2)$ in deep Euclidean region ↔ matching scale and scheme dependence
- ⋆ Numerics: constraints from τ data $(\rho(s) = \frac{1}{\pi}{\rm Im}\Pi_{LR}(s))$ plus { QCD chiral sum rules / QCD duality (FESR) }

Electroweak Penguins at leading chiral order (p^0): comparisons

$\langle(\pi\pi)_{I=2}|Q_7(2\,\mathrm{GeV})|K\rangle^{\overline{MS}\text{-NDR}}$ (GeV^3)



$\langle(\pi\pi)_{I=2}|Q_8(2\,\mathrm{GeV})|K\rangle^{\overline{MS}\text{-NDR}}$ (GeV^3)



- ⋆ DISPERSIVE: Narison, BGP, CDGM
- ⋆ LATTICE: RBC, CP-PACS, $SPQ_{\mathrm{CD}}R$ ($K \to \pi\pi$)
  - extrapolated to chiral limit
  - quenched approximation
  - quoted error is statistical only

Electroweak Penguins: chiral corrections

⋆ Lattice result at NLO in ChPT ($SPQ_{\rm CD}R$, $K \to \pi\pi$):

$$\langle (\pi\pi)_{I=2} | Q_7 (2\,\mathrm{GeV}) | K \rangle^{\overline{MS}\text{-NDR}} = (0.14 \pm 0.02)\ \mathrm{GeV}^3$$
$$\langle (\pi\pi)_{I=2} | Q_8 (2\,\mathrm{GeV}) | K \rangle^{\overline{MS}\text{-NDR}} = (0.69 \pm 0.12)\ \mathrm{GeV}^3$$

⋆ Dispersive result at NLO in ChPT
(CDGM + chiral loops + conservative bound NLO LECs)

$$\langle (\pi\pi)_{I=2} | Q_7 (2\,\mathrm{GeV}) | K \rangle^{\overline{MS}\text{-NDR}} = (0.16 \pm 0.06)\ \mathrm{GeV}^3$$
$$\langle (\pi\pi)_{I=2} | Q_8 (2\,\mathrm{GeV}) | K \rangle^{\overline{MS}\text{-NDR}} = (1.10 \pm 0.36)\ \mathrm{GeV}^3$$

⋆ Reasonable agreement $\Rightarrow$ reasonably solid prediction

Gluonic Penguins at leading chiral order (p^2) HPR

$$Q_6(\mu) \;\rightarrow\; \langle \lambda D_\mu U^\dagger D_\mu U \rangle \left\{ \frac{-16 L_5 \langle \bar{q} q \rangle^2}{F^6} + \frac{8 n_f \mu^{4-d}}{(4\pi)^{d/2} \Gamma(d/2) F^4} \int_0^\infty dQ^2\, Q^{d-2}\, \mathcal{W}(Q^2) \right\}_{\overline{\mathrm{MS}}}$$

$$\mathcal{W}(Q^2) \;\leftrightarrow\; \text{Transverse part of } \langle 0 | T\left((\bar{s}_L q_R)\,(\bar{q}_L d_R)\,(\bar{d}_R \gamma^\mu u_R)\,(\bar{u}_R \gamma^\nu s_R) \right) |0\rangle$$

$\mathcal{W}(Q^2)$

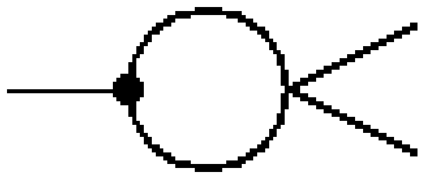
Low Q^2 $\rightarrow$ ChPT

High Q^2 $\rightarrow$ OPE (leading term) + factorization of local operators

Intermediate Q^2 $\rightarrow$ Interpolation with meromorphic function (QCD_∞), satisfying both long- and short- distance constraints

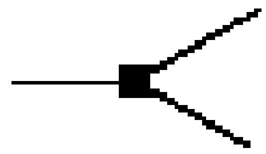
- ⋆ Large corrections to factorization ($B_6^{1/2}(1\mathrm{GeV}) \sim 3$)
- ⋆ Correlated enhancement of $\mathrm{Re}A_0$, through $\langle Q_2 \rangle_0$
- ⋆ In principle the method is improvable
- ⋆ Similar results found within ENJL model, not within lattice

Going beyond leading chiral order (FSI)



Loops dominated by infrared log M_π: NLO in $1/N_c$

Model independent



NLO ChPT LECs fixed at LO in $1/N_c$

Major uncertainty

- ◇ Sizeable enhancement of $\langle Q_6 \rangle_0$ ($\sim 40\%$): $\chi_I \neq 0$, but still $\chi_0 < \chi_0^{\rm expt}$

  [important step towards a complete calculation]
- ◇ However, dispersive part of the amplitude is still quite uncertain
- ◇ Only consistent way to include FSI is by working at a given order in ChPT
- ◇ Lattice QCD: presently works at LO and NLO in ChPT

  [Possibility to fully reconstruct FSI phases from Euclidean simulations in finite volume]

Isospin violation in ϵ': $\Omega_{\rm eff}$ CENP

Collect all effects to first order in α and $(m_u - m_d)$:

$$\begin{aligned}
\epsilon' &= \frac{-ie^{i(\chi_2-\chi_0)}}{\sqrt{2}}\,\omega\,\left[\frac{{\rm Im}A_0}{{\rm Re}A_0} - \frac{{\rm Im}A_2}{{\rm Re}A_2}\right] \\
&= \frac{-ie^{i(\chi_2-\chi_0)}}{\sqrt{2}}\,\omega_+\,\left[\frac{{\rm Im}A_0^{(0)}}{{\rm Re}A_0^{(0)}}(1+\Delta_0+f_{5/2}) - \frac{{\rm Im}A_2}{{\rm Re}A_2^{(0)}}\right] \\
&= \frac{-ie^{i(\chi_2-\chi_0)}}{\sqrt{2}}\,\omega_+\,\left[\frac{{\rm Im}A_0^{(0)}}{{\rm Re}A_0^{(0)}}(1-\Omega_{\rm eff}) - \frac{{\rm Im}A_2^{\rm emp}}{{\rm Re}A_2^{(0)}}\right]
\end{aligned}$$

$$\Omega_{\rm eff} = \Omega_{\rm IB} - f_{5/2} - \Delta_0$$

1. Effect of $\Delta I = 5/2$ amplitude ($O(\alpha)$): $\omega = \frac{{\rm Re}A_2}{{\rm Re}A_0} = \omega_+\,(1+f_{5/2})$; $\quad \omega_+ = \left.\frac{{\rm Re}A_2^+}{{\rm Re}A_0}\right|_{\rm exp}$
2. IB in ReA_0 and ImA_0 (mainly $O(\alpha)$): $\Delta_0 = \frac{{\rm Re}A_0^{(0)}}{{\rm Re}A_0}\cdot\frac{{\rm Im}A_0}{{\rm Im}A_0^{(0)}} - 1$
3. Q_6 effect in ImA_2 ($O(\alpha, m_u - m_d)$): $\Omega_{\rm IB} = \frac{{\rm Re}A_0^{(0)}}{{\rm Re}A_2^{(0)}}\cdot\frac{{\rm Im}A_2^{\rm non-emp}}{{\rm Im}A_0^{(0)}}$

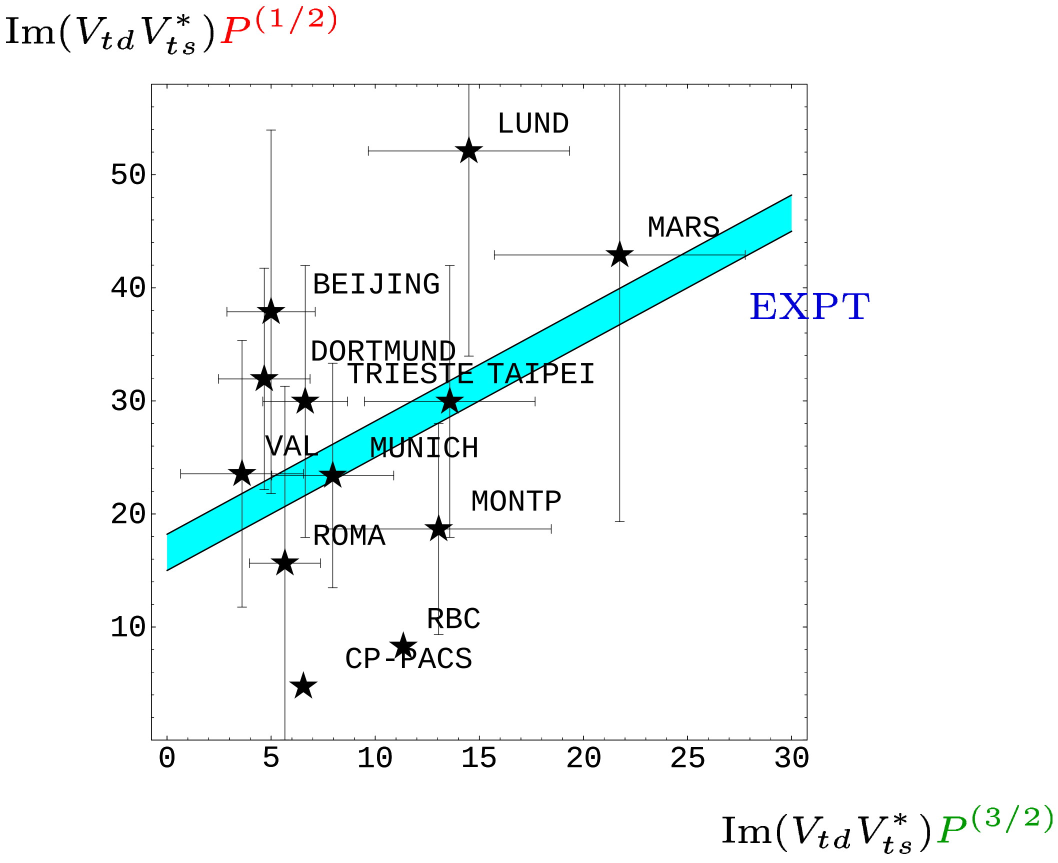
Isospin violation in ϵ': results

	$\alpha = 0$		$\alpha \neq 0$	
	LO	NLO	LO	NLO
$10^2 \times \Omega_{\rm IB}$	11.7	15.9 ± 4.5	18.0 ± 6.5	22.7 ± 7.6
$10^2 \times \Delta_0$	-0.004	-0.41 ± 0.05	8.7 ± 3.0	8.3 ± 3.6
$10^2 \times f_{5/2}$	0	0	0	8.3 ± 2.4
$10^2 \times \Omega_{\rm eff}$	11.7	16.3 ± 4.5	9.3 ± 5.8	6.0 ± 8.0

- ⋆ Results based on a complete calculation at NLO in ChPT
- ⋆ NLO LECs estimated within factorization (leading term in $1/N_c$): need as input strong LECs up to $O(p^6)$ [delicate point]
- ⋆ Errors: (i) propagation from input parameters (correlated); (ii) estimate of missing non-factorizable effects (both at LO and NLO)

Present estimates of ϵ'/ϵ

$$\left|\frac{\epsilon'}{\epsilon}\right| \simeq \mathrm{Im}(V_{td}V_{ts}^*) \cdot \left[\underbrace{r\, y_6(\mu)\langle Q_6(\mu)\rangle_0\,(1-\Omega_{\rm eff})}_{P^{(1/2)}} - \underbrace{\frac{r}{\omega_+}\, y_8(\mu)\langle Q_8(\mu)\rangle_2}_{P^{(3/2)}}\right]$$



* $\langle Q_i\rangle_I$ from original calculations
* Updated values of $\Omega_{\rm IB}$, $\Omega_{\rm eff}$
* Correlations not taken into account
* Several calculations consistent with expt.
  Key ingredient: NLO in ChPT and $1/N_c$

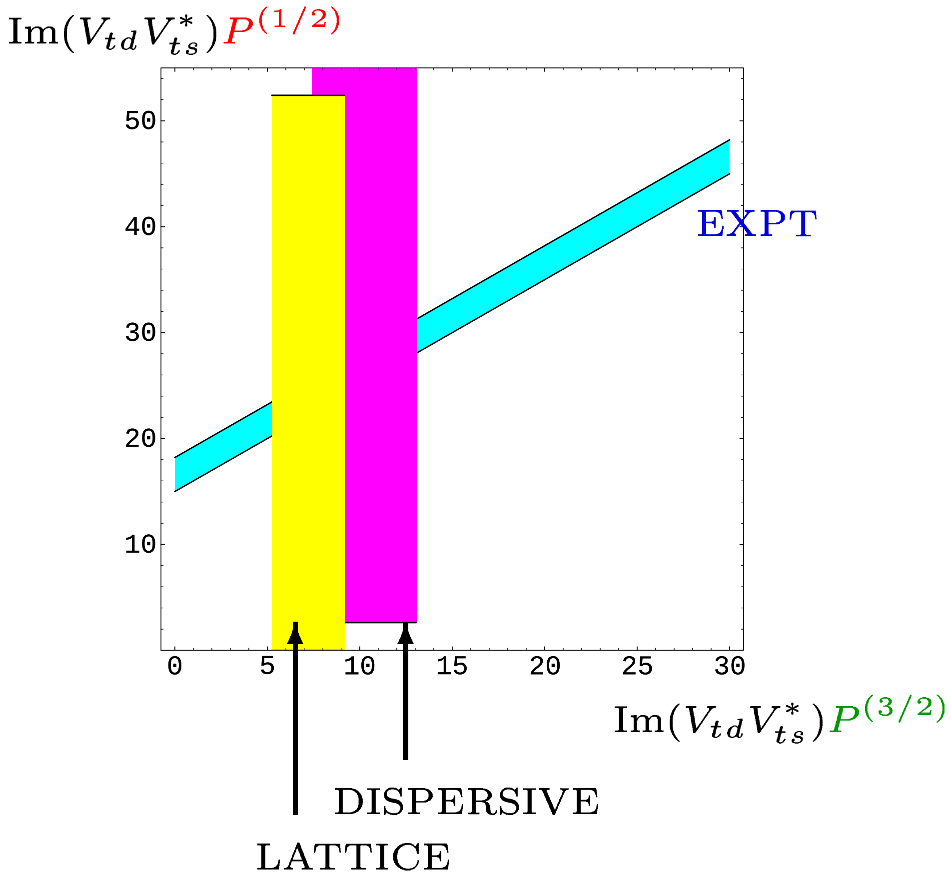
Phenomenology: where are we, really ?

Model-independent range for $\langle Q_8 \rangle_2$:

$\rightarrow$ Reasonable convergence of Lattice and Dispersive calculations

$\rightarrow$ Beyond factorization at $O(p^0)$

$\rightarrow$ Contains effect of chiral corrections



$*$ To reproduce expt. within SM, need:

$$\langle Q_6(2\mathrm{GeV})\rangle_0^{\mathrm{NDR}} \sim (-0.75 \pm 0.15)\,\mathrm{GeV}^3$$

$$B_6^{(1/2),\mathrm{NDR}}(2\,\mathrm{GeV}) \sim 1.26 \pm 0.25$$

$*$ Calculations of $\langle Q_6 \rangle_0$ far from converging

Conclusion

- ◇ Several calculations come close to expt.: but most predictions are not solid
  [Several uncontrolled approximations]

- ◇ ϵ'/ϵ not (yet) a quantitative test of the SM

- ◇ Considerable improvement of lattice and analytic calculations in last few years $\Rightarrow$
  Reasonable agreement on $\langle Q_8 \rangle_2$, IB effects not as large as thought before

- ◇ Challenge: $\langle Q_6 \rangle_0$ beyond factorization
  - * Analytic approaches: control $1/N_c$ corrections at LO and NLO in chiral expansion (including scheme dependence)
  - * Lattice: go beyond leading chiral order (DWF), control quenching approximation, direct calculation of $\langle (\pi\pi)_{I=0} | Q_6 | K \rangle$, ...
  - * In this process analytic methods can serve as quantitative “test” for lattice