CKM angles from non-leptonic B decays using $SU(3)$ flavour symmetry

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R. Fleischer and J. M., Phys.Rev.D66:054009,2002
R. Fleischer, G. Isidori, J. M. , JHEP 0305:053,2003

⇒ B physics is one of the most fertile grounds at present to test SM and to look for first signals of New Physics.

⇒ A new and successful *era of precision* just started:

- e^+e^- **B Factories**: **Babar** in PEP II (SLAC), **Belle** in KeK-B (KEK), **CLEO** (Cornell).
- Also the 2^{nd} generation of experiments with the hadronic machines **LHCb**, **BTeV** and the future **SuperBabar or SuperBelle**.

- Determination of $\sin\phi_d$ from $\mathcal{A}_{\rm CP}^{mix}(B_d \to J/\Psi K_S)$ giving a world average of 0.734 ± 0.054.
  In SM $\phi_d = 2\beta$, in presence of NP ⇒ $\phi_d = 2\beta + \delta^{NP}$.

- Non-leptonic B decays:

$$B \to \pi K\,,\; B_d \to \pi\;\pi,\; B_s \to K\;\;K$$

  play a fundamental role to determine γ and to search for New Physics.

  – **Keypoint**: How to deal with hadronic matrix elements and to control penguin contributions.

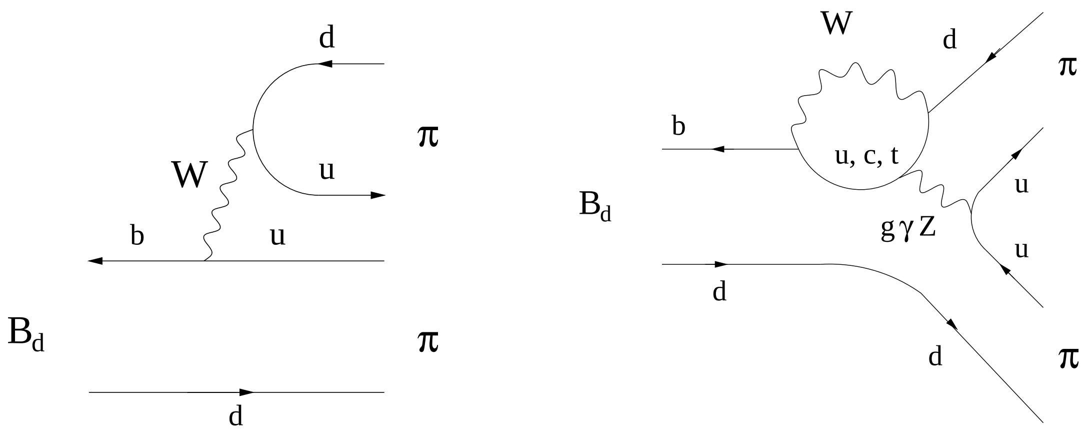
  – Two different/complementary **approaches**:

    ∗ Evaluation of some hadronic parameters directly from QCD: *QCD Factorization* or *PQCD*.

    ∗ Use of flavour symmetries to extract the maximal possible information from data to control hadronic parameters.

$B_d \to \pi^+\pi^-$ decay

STEP 1 Feynman diagrams for $B_d \to \pi^+\pi^-$ in SM:



Tree diagrams color allowed and QCD/EW penguins.

General Amplitude Parametrization in SM:

$$A(B_d^0 \to \pi^+\pi^-) = \lambda_u^{(d)} \left(A_{\rm CC}^u + A_{\rm pen}^{(u)} \right) + \lambda_c^{(d)} A_{\rm pen}^{(c)} + \lambda_t^{(d)} A_{\rm pen}^{(t)}$$

$$= \mathcal{C} \left(e^{i\gamma} - \mathbf{d} e^{i\theta} \right)$$

where all hadronic information is collected in:

$$\mathbf{d} e^{i\theta} \equiv \frac{1}{R_b} \left(\frac{A_{\rm pen}^{ct}}{A_{\rm CC}^u + A_{\rm pen}^{ut}} \right) \qquad \mathcal{C} \equiv \lambda^3 A\, R_b \left(A_{\rm CC}^u + A_{\rm pen}^{ut} \right)$$

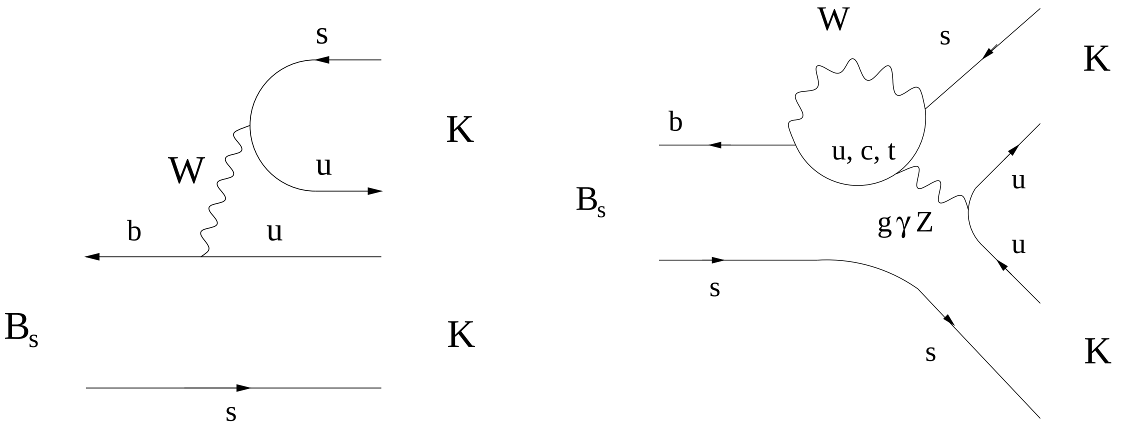
STEP 2 Construction of CP asymmetries for $B_d \to \pi^+\pi^-$:

$$\mathcal{A}_{\rm CP}^{\rm dir} = {\rm func}(\mathbf{d}, \theta, \gamma) \qquad \mathcal{A}_{\rm CP}^{\rm mix} = {\rm func}(\mathbf{d}, \theta, \gamma, \phi_d)$$

⇒ Counting of parameters: **Hadronic parameters:** $\mathbf{d}$, θ. **Weak parameters:** ϕ_d and CKM angle γ.

STEP 3 There is a closely related process $B_s \to KK$:

Feynman diagrams entering this process are:



$$A(B_s^0 \to K^+K^-) = \left(\frac{\lambda}{1-\lambda^2/2}\right) \mathcal{C}' \left[e^{i\gamma} + \left(\frac{1-\lambda^2}{\lambda^2}\right) \mathbf{d}' e^{i\theta'}\right]$$

$$\mathbf{d'e^{i\theta'}} \equiv \frac{1}{R_b}\left(\frac{A_{\text{pen}}^{ct\,\prime}}{A_{\text{CC}}^{u\,\prime} + A_{\text{pen}}^{ut\,\prime}}\right) \quad \mathcal{C}' \equiv \lambda^3 A\, R_b \left(A_{\text{CC}}^{u\,\prime} + A_{\text{pen}}^{ut\,\prime}\right)$$

Chosen to have same functional dependence on penguins except for the interchange $d \leftrightarrow s$

$\Rightarrow$ Its corresponding CP asymmetries for $B_s \to K^+K^-$:

$$\mathcal{A}_{\text{CP}}^{\text{dir}} = \text{func}(\mathbf{d}', \theta', \gamma) \quad \mathcal{A}_{\text{CP}}^{\text{mix}} = \text{func}(\mathbf{d}', \theta', \gamma, \phi_s)$$

$\Rightarrow$ Counting of parameters: **Hadronic parameters:** $\mathbf{d}'$, θ'. **Weak parameters:** ϕ_s and CKM angle γ.

In SM weak mixing parameter: $\phi_s = -2\lambda^2\eta \sim 0$

STEP4 BOTH processes and their parameters can be related via U-spin symmetry

Observables: $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi\pi), \mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi\pi),$ $\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to KK), \mathcal{A}_{\rm CP}^{\rm mix}(B_s \to KK)$

Parameters are reduced via U-spin: $\mathbf{d}e^{i\theta} = \mathbf{d}'e^{i\theta'}$

γ, ϕ_d, ϕ_s / $\mathbf{d}, \theta, \mathbf{d}', \theta'$ $\Rightarrow$ γ, ϕ_d / $\mathbf{d}, \theta$ $\quad \phi_s(B_s \to J/\Psi\phi)$

Tests of U-spin breaking:

$\Rightarrow$ Once data from $B_s \to KK$ and $B_s \to J/\Psi\phi$ (to determine ϕ_s) will be available, taken ϕ_d from $B_d \to J/\Psi K_S$ we will have only three unknowns.

$\Rightarrow$ Already now one can define $\xi = \mathbf{d}'/\mathbf{d}$ and $\Delta\theta = \theta' - \theta$ and test the sensitivity of the results.

STEP5 Moreover **d** is NOT a fully free parameter, we can constrain (substitute) it using an observable called H:

$$H \equiv \frac{1}{\epsilon}\left|\frac{\mathcal{C}'}{\mathcal{C}}\right|^2 \left[\frac{M_{B_d}}{M_{B_s}} \frac{\Phi(\frac{M_K}{M_{B_s}},\frac{M_K}{M_{B_s}})}{\Phi(\frac{M_\pi}{M_{B_d}},\frac{M_\pi}{M_{B_d}})} \frac{\tau_{B_s}}{\tau_{B_d}}\right] \left[\frac{\mathrm{BR}(B_d\to\pi^+\pi^-)}{\mathrm{BR}(B_s\to K^+K^-)}\right]$$

H has the peculiar property that in terms of parameters:

$$H = \frac{1 - 2\mathbf{d}\cos\theta\cos\gamma + \mathbf{d}^2}{\epsilon^2 + 2\epsilon\mathbf{d}'\cos\theta'\cos\gamma + \mathbf{d}'^2}$$

Data on $B_s \to KK$ is coming soon....Meanwhile

CONTACT with B-Factories: $B_d \to \pi^\pm K^\mp$

$B_d \to \pi^\pm K^\mp$ and $B_s \to K^+ K^-$ differ in their spectator quarks, this means:

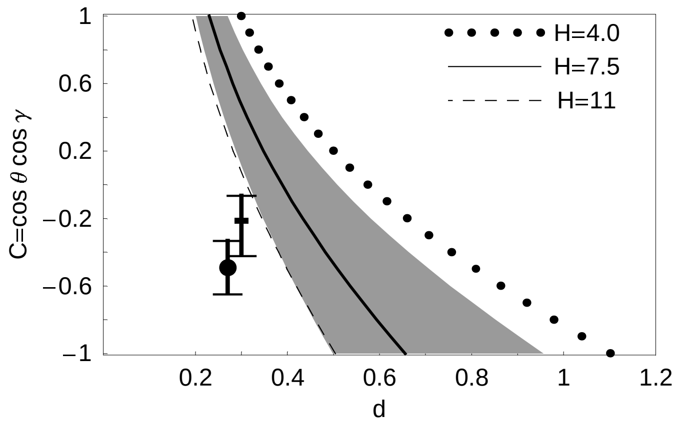
$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) \approx \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^\mp K^\pm)$$

$$\text{BR}(B_s \to K^+K^-) \approx \text{BR}(B_d \to \pi^\mp K^\pm) \frac{\tau_{B_s}}{\tau_{B_d}}$$

yielding

$$H \approx \frac{1}{\epsilon}\left(\frac{f_K}{f_\pi}\right)^2 \left[\frac{\text{BR}(B_d \to \pi^+\pi^-)}{\text{BR}(B_d \to \pi^\mp K^\pm)}\right] = 7.5 \pm 0.9$$

a) H and $-1 \leq \cos\theta\cos\gamma \leq 1 \Rightarrow 0.2 \lesssim \mathbf{d} \lesssim 1.$



b) Alternatively $\mathbf{d} = f(H, \theta, \gamma;\ \xi, \Delta\theta)$

Of course, once $B_s \to KK$ data will be available WE WILL NOT NEED this hypothesis and only U-spin breaking effects will be important.

STEP6 EXPLORING THE ALLOWED REGION in $B_d \to \pi\pi$ to the SM.

Starting point is the general expression:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) = \mp \left[\frac{\sqrt{4d^2 - (u + vd^2)^2}\,\sin\gamma}{(1 - u\cos\gamma) + (1 - v\cos\gamma)d^2} \right]$$

where

$u, v, d = F_i(\mathcal{A}_{\rm CP}^{\rm mix}, H, \gamma, \phi_d(B_d \to J/\Psi K_s); \xi, \Delta\theta)$.

- We start in the U-spin limit ($\xi = 1, \Delta\theta = 0$)
- Important symmetry:

$$\phi_d \to 180^\circ - \phi_d \quad \gamma \to 180^\circ - \gamma$$

- Using present world average $\sin\phi_d = 0.734 \pm 0.054,$ one gets two solutions

$$\phi_d = \left(47^{+5}_{-4}\right)^\circ \vee \left(133^{+4}_{-5}\right)^\circ.$$

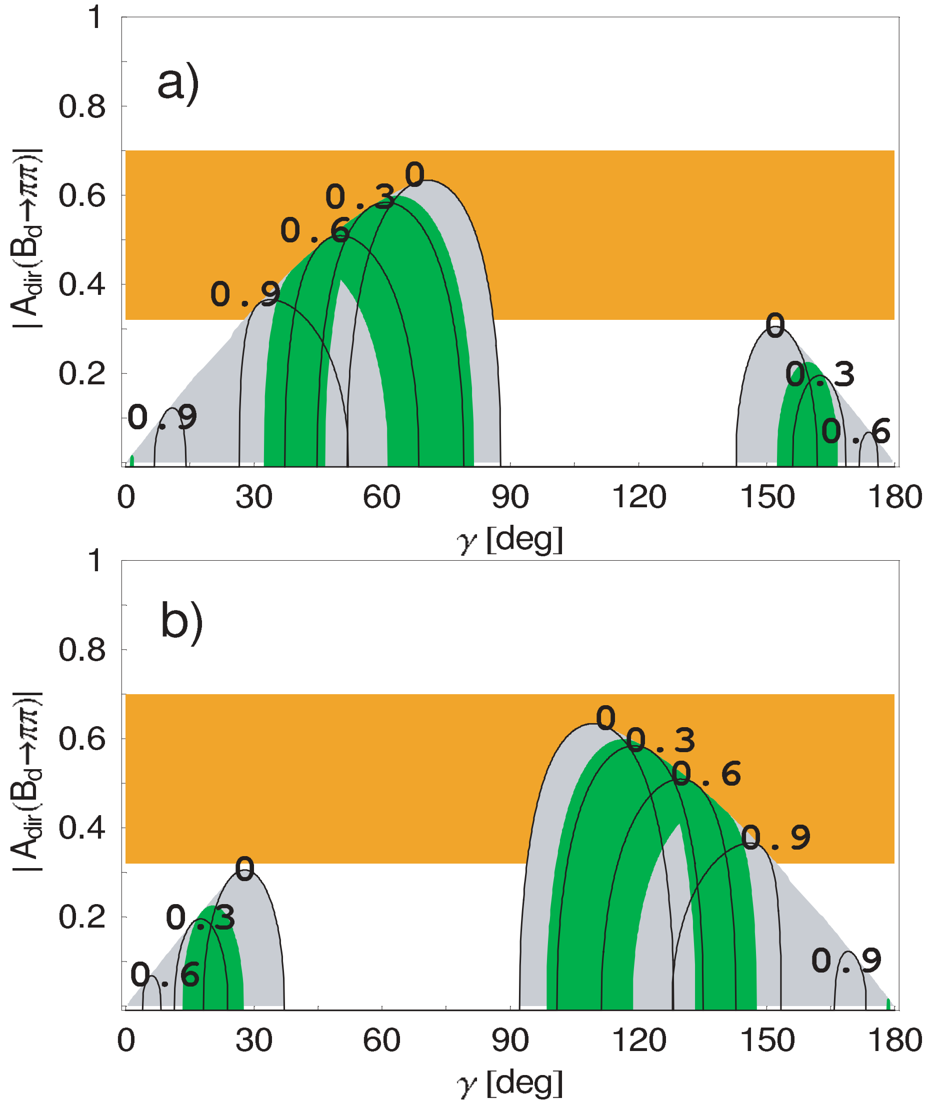
This approach allow us to explore BOTH solutions, $\cos\phi_d > 0$ and $\cos\phi_d < 0$.

Experimental situation uncertain, present naive average is (including PDG enlarged errors):

$$\begin{aligned}
\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-) &= -0.51 \pm 0.19\ (0.23)\\
\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-) &= +0.49 \pm 0.27\ (0.61)
\end{aligned}$$

Determination of γ ($\mathcal{A}_{\rm CP}^{\rm mix} > 0$)

We vary $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ for all positive values, H=7.5 and the two solutions for ϕ_d:



Orange band: Experimental range of $\mathcal{A}_{\rm CP}^{\rm dir}$
Green band: Experimental range of $\mathcal{A}_{\rm CP}^{\rm mix}$

a) corresponds to $\phi_d = 47^\circ$ and the prediction for γ is

$$35^\circ \lesssim \gamma \lesssim 79^\circ$$

b) corresponds to $\phi_d = 133^\circ$ and the prediction for γ is

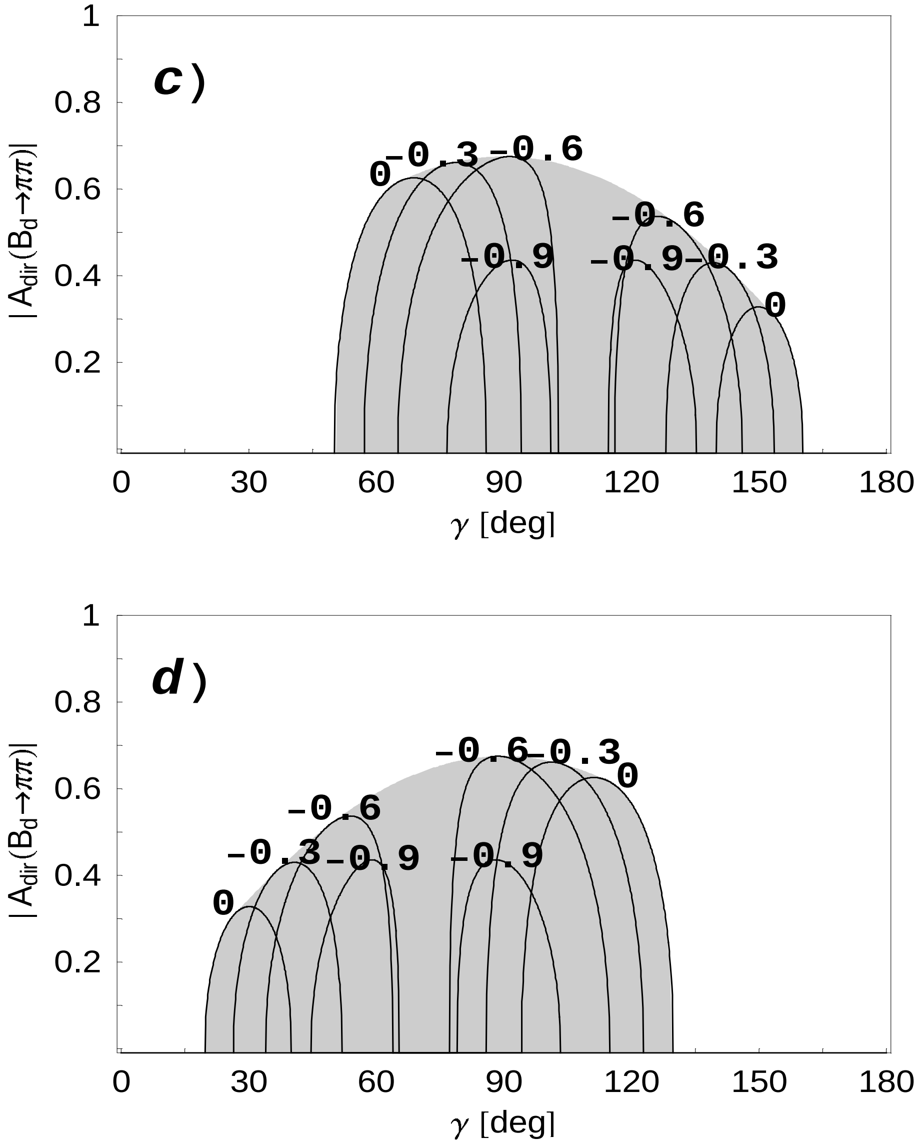
$$101^\circ \lesssim \gamma \lesssim 145^\circ$$

- The two plots are a manifestation of the symmetry.
- Excluded regions are:

$$88^\circ \lesssim \gamma \lesssim 143^\circ \text{ (for } \phi_d = 47^\circ)$$
$$37^\circ \lesssim \gamma \lesssim 92^\circ \text{ (for } \phi_d = 133^\circ)$$

Determination of γ ($\mathcal{A}_{\rm CP}^{\rm mix} < 0$)

If $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$ is varied for all negative values, H=7.5 and the two solutions for ϕ_d:



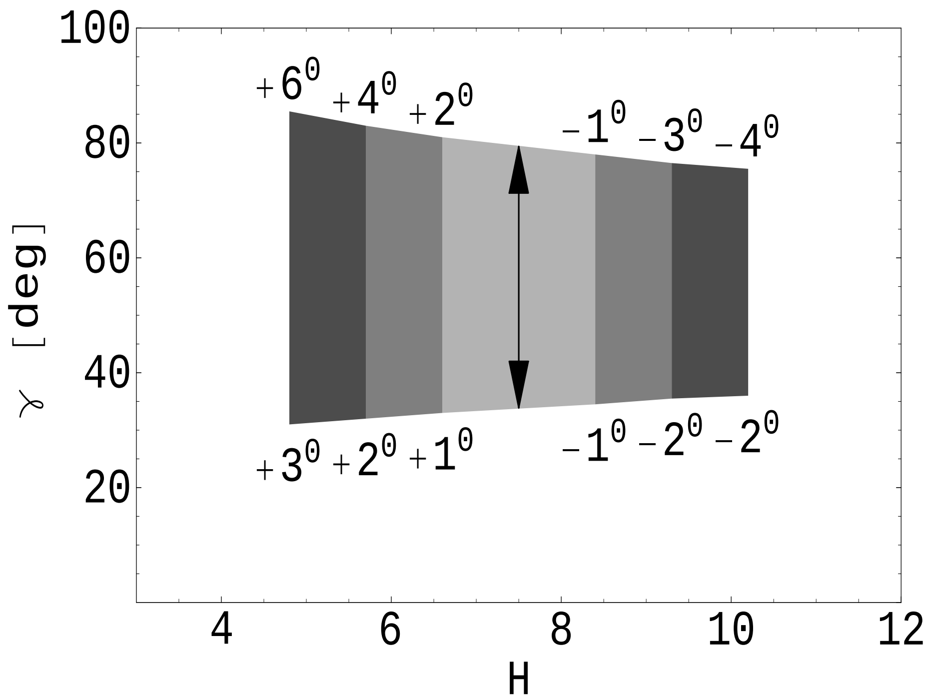
The situation is not as interesting, the gap dissappears and the predicted region for γ becomes too wide.

Sensitivity to H and ξ, $\Delta\theta$

Uncertainty associated to the variation of the different hadronic parameters.

- $\boxed{\text{H}} \approx \frac{1}{\epsilon}\left(\frac{f_K}{f_\pi}\right)^2 \left[\frac{\text{BR}(B_d\to\pi^+\pi^-)}{\text{BR}(B_d\to\pi^\mp K^\pm)}\right]$ (B-factories)
  - $\phi_d = 47^\circ$ and H inside experimental range:

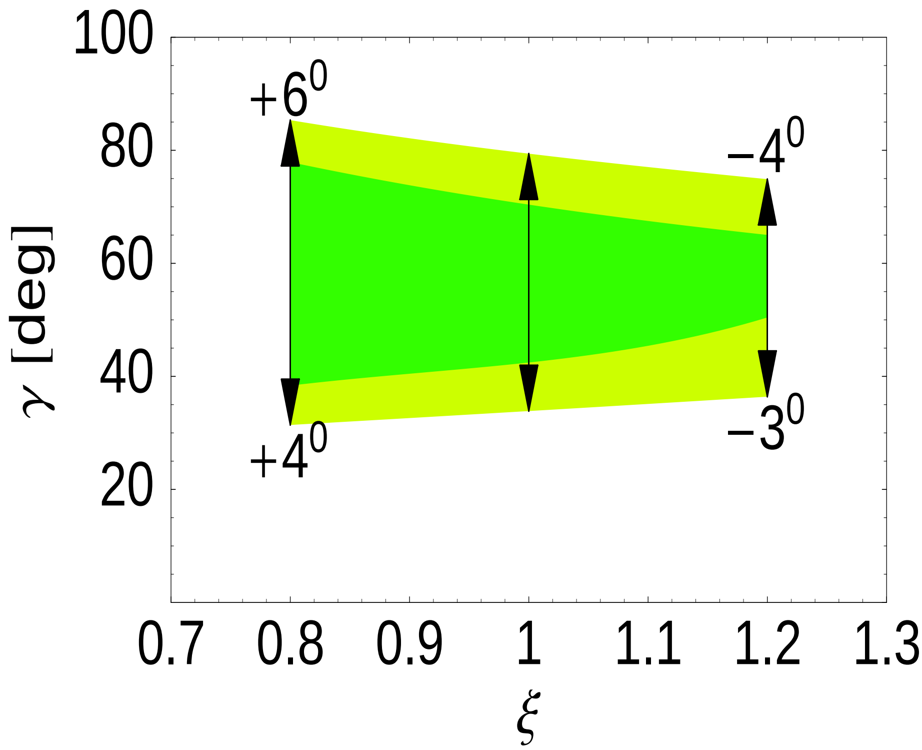
$$H = 7.5 \pm 0.9.$$



  - There is a symmetric solution with $\phi_d = 133^\circ$ and $\mathcal{A}_{\text{CP}}^{\text{mix}} > 0$ with $\gamma > 90^\circ$.
  - H dependence very mild at 1 σ, error induced is $\sim \pm 2^\circ$.
  - Variation of the range at 1 σ,2 σ,3 σ, to be conservative and include the uncertainty associated to *spectator quark hypothesis*.
  - Larger values of H reduces the range for γ.
  - Uncertainty will be drastically reduced with data from $B_s \to KK$, only U-spin.

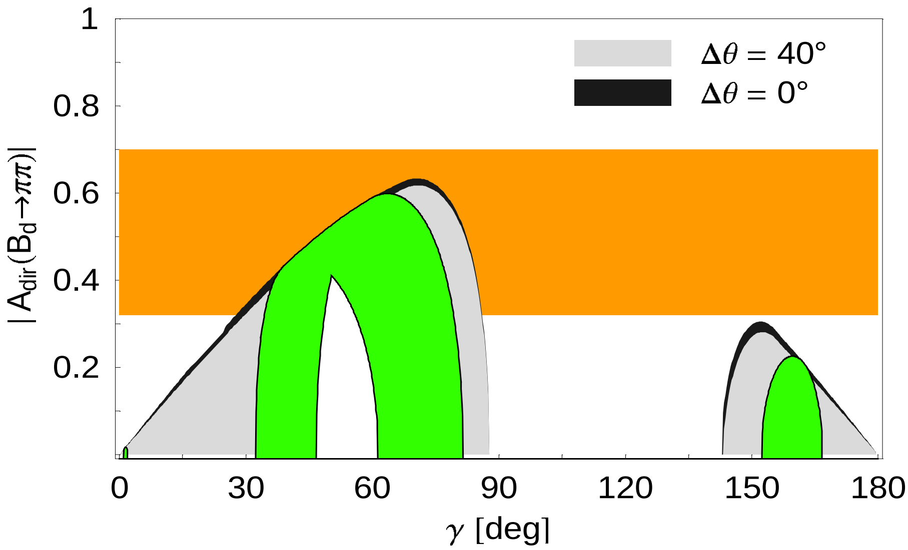
U-spin breaking parameters

- $\boxed{\xi} = \mathbf{d'/d}$. This is the most important uncertainty, we allow for a large U-spin breaking: $\xi \in [0.8, 1.2]$



  - This is the solution with $\phi_d = 47°$ and $\mathcal{A}_{\rm CP}^{\rm mix} > 0$. It exists a symmetrical solution with $\phi_d = 133°$.
  - Larger values of ξ imply stronger constraints on γ.
  - Error in the determination of γ is about $\pm 5°$.
  - Sensitivity to:
    * experimental result $\Rightarrow$ reduction of error bars to 1/3 $\Rightarrow$ (inner green area) implies sizeable reduction to the range for γ.
    * $\Delta\theta$ sensitivity is very small. Its influence on the determination of γ (up to $40°$) seems negligibly small.

- $\boxed{\Delta\theta}$ Here we allow for a large breaking $\Delta\theta = 40^\circ$:
  the influence on the determination of γ
  is negligibly small.



– Allowing for $\Delta\theta = +40^\circ$ and $\xi = 0.8, 1, 1.2$ the error induced for each value of ξ is *at most 1 degree*.

Largest uncertainty on the determination on γ comes from U-spin breaking parameter ξ

$B_s \to K\ K$ decay

If we eliminate θ' we obtain:

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) = \pm \left[\frac{\sqrt{4\tilde{d}'^2 - \left(u' + v'\tilde{d}'^2\right)^2}\ \sin\gamma}{(1 - u'\cos\gamma) + (1 - v'\cos\gamma)\tilde{d}'^2} \right]$$

where $u', v', \tilde{d}' = f(H, \mathcal{A}_{\rm CP}^{\rm mix}, \gamma, \phi_s(B_s \to J/\Psi\phi); \xi, \Delta\theta)$

- Measurement of CP-averaged $\mathrm{BR}(B_s \to KK) \Rightarrow$ cleaner determination of H.
- Two hadronic parameters $(d', \theta') \Rightarrow H, \mathcal{A}_{\rm CP}^{\rm mix}$.
- Two weak phases $\Rightarrow \gamma$, ϕ_s

Differences with $B_d \to \pi^+\pi^-$:

- Run II Tevatron and LHCb
- No weak phase ambiguity, NP in $\phi_s \Rightarrow$ large shift.
- Accidental supression due to ϵ.

$$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-) = -\epsilon\xi H \mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$$

- SM range is strongly restricted.
- Sensitivity to U-spin breaking very small.
- Extra observable $\mathcal{A}_{\Delta\Gamma}$:coefficient of $\sinh(\Delta\Gamma_s t/2)$ of the time-dependent CP asymmetry. It can be extracted from "untagged" rate thanks to sizeable $\Delta\Gamma_s$.

Allowed region in the $\mathcal{A}_{\rm CP}^{\rm mix}(B_s \to K^+K^-)$–$\mathcal{A}_{\rm CP}^{\rm dir}(B_s \to K^+K^-)$ plane:

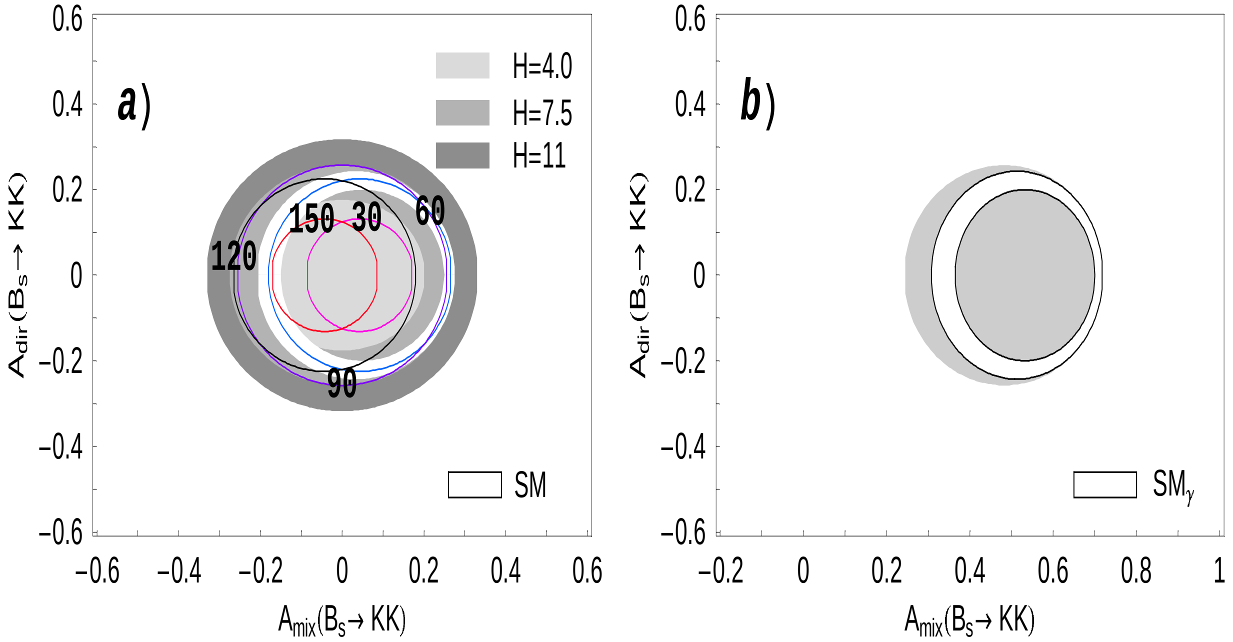


Figure 1: a) $\phi_s = 0^o$ and b) $\phi_s = 30^o$ (new physics).
SM region is very constrained.

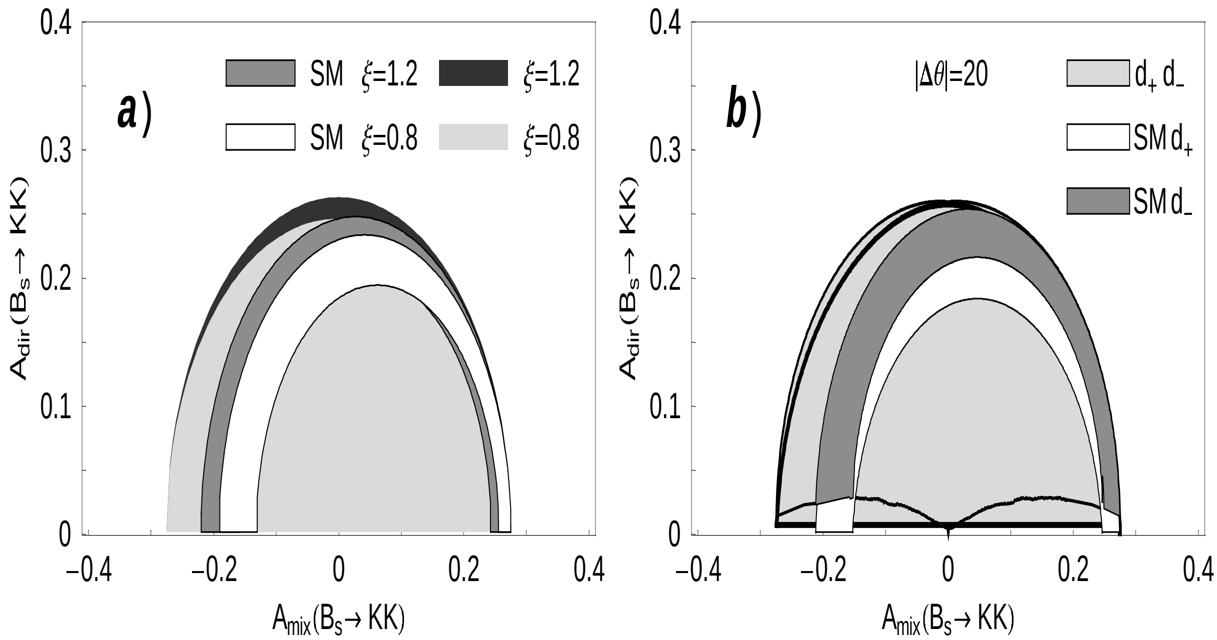


Figure 2: Impact of variations of (a) $\xi \in [0.8, 1.2]$, and
(b) $\Delta\theta \in [-20^\circ, +20^\circ]$ on the allowed region.

Sensitivity to U-spin breaking is very small $\Rightarrow$ robust prediction.

How can we determine α and β even in presence of certain type of New Physics?

We will use two observables:

- $R_b \equiv \left|\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right|$
  - exclusive/inclusive transitions $b \to u\ell\overline{\nu}_\ell$ and $b \to c\ell\overline{\nu}_\ell$
  - No New Physics effects expected.

  From $R_b^{\max} = 0.46 \Rightarrow$ $|\beta|_{\max} = 27^\circ$

- γ from $\mathcal{A}_{\rm CP}^{\rm dir}(B_d \to \pi^+\pi^-)$ and $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to \pi^+\pi^-)$
  - No New Physics in $\Delta B = 1$, $\Delta S = 1$ decay amplitudes.
  - Generic New Physics in B_d^0–$\overline{B_d^0}$ mixing.

$\Rightarrow$ Concerning $\sin\phi_d$ from $\mathcal{A}_{\rm CP}^{\rm mix}(B_d \to J/\psi K_{\rm S})$ it implies in general:

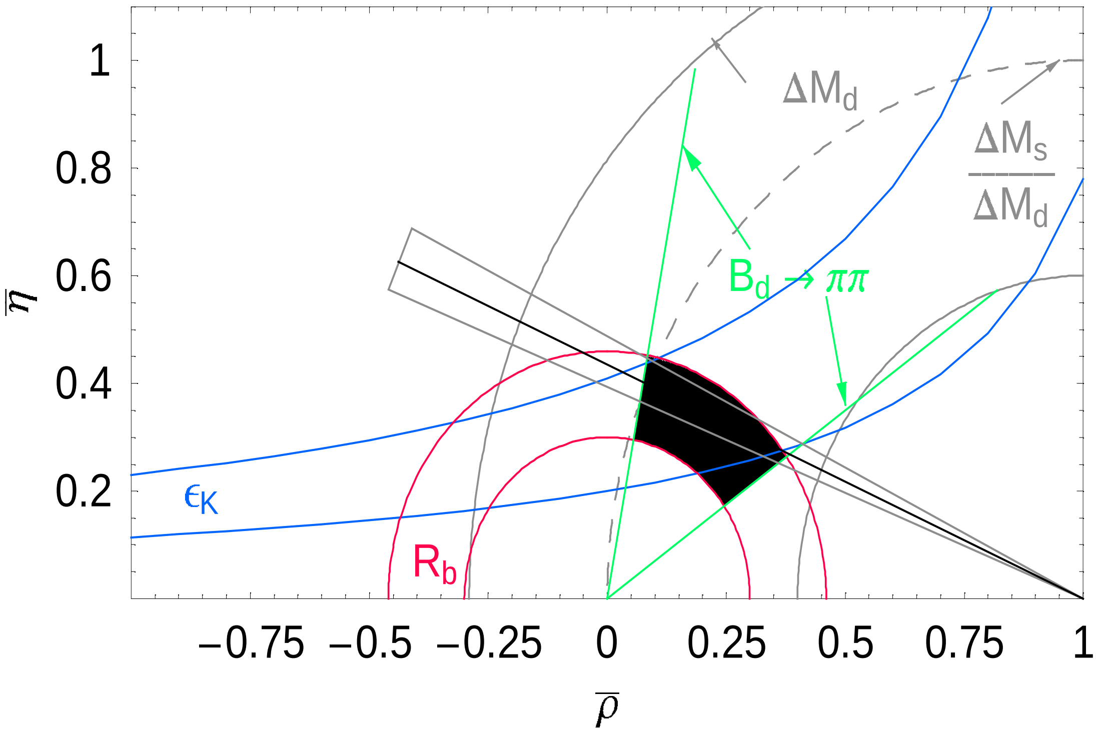
$$\phi_d = \phi_d^{\rm SM} + \phi_d^{\rm NP} = 2\beta + \phi_d^{\rm NP}$$

so we cannot use it to determine β directly in presence of New Physics, but it is used as an input for the CP-asymmetries of $B_d \to \pi\pi$.

$\Rightarrow$ Also ΔM_d and $\Delta M_s/\Delta M_d$ cannot be used in presence of New Physics to determine the side R_t.

Two possible scenarios

a) Case with $\phi_d = 47^\circ$



– Only using R_b and CP-asymmetry of $B_d \to \pi^+\pi^-$ (including ϕ_d)

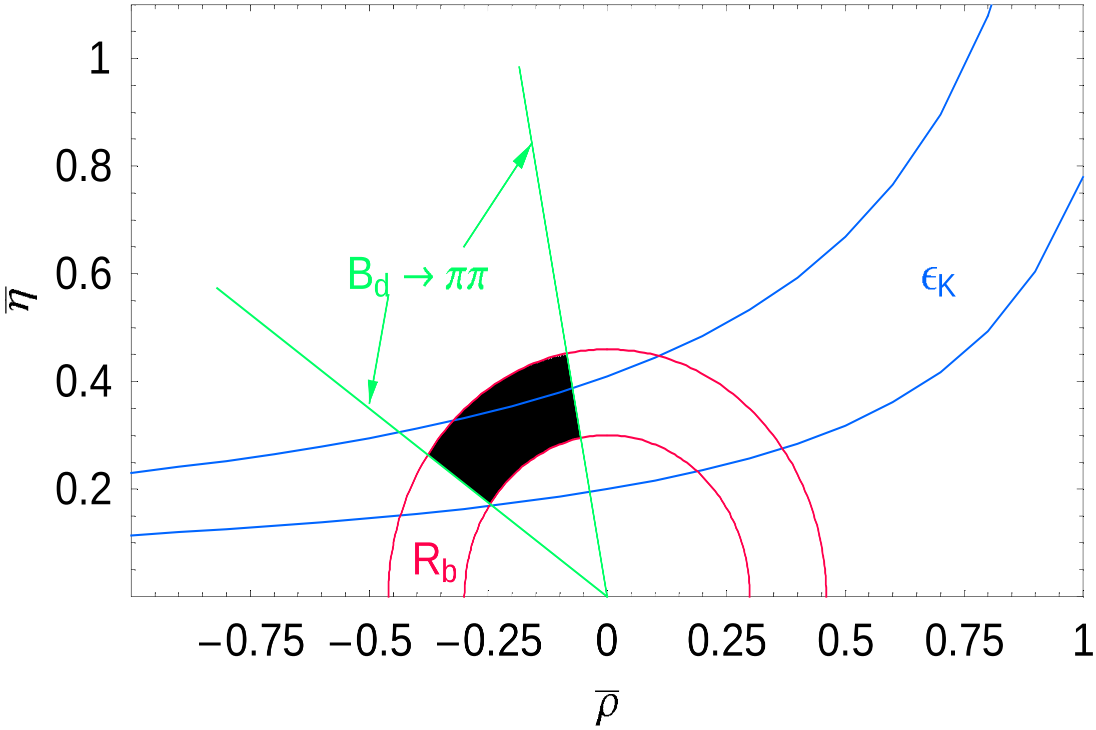
– Good agreement with SM interpretation of ϕ_d from $B_d \to J/\psi K_S$, ΔM_d, $\Delta M_s/\Delta M_d$ and ϵ_K.

Case of $\xi = 1$	α	β	γ
$\phi_d = 47^\circ$	$(103 \pm 29)^\circ$	$(20 \pm 7)^\circ$	$(57 \pm 22)^\circ$

Error associated to $\xi \in [0.8, 1.2]$:

$$\Delta\alpha = \pm 4^\circ \qquad \Delta\beta = \pm 1^\circ \qquad \Delta\gamma = \pm 5^\circ$$

b) Case with $\phi_d = 133°$, NEW PHYSICS is required.



- γ is in the second quadrant
- β is indeed smaller than in the $\phi_d = 47°$ case.
- still consistent with the ϵ_K hyperbola.
- several models (in particular supersymmetry) can generate this second solution.

Case of $\xi = 1$	α	β	γ
$\phi_d = 133°$	$(44 \pm 20)°$	$(15 \pm 7)°$	$(123 \pm 22)°$

Error associated to $\xi \in [0.8, 1.2]$:

$$\Delta\alpha = \pm 4° \qquad \Delta\beta = \pm 1° \qquad \Delta\gamma = \pm 5°$$

Conclusions

- We have proposed a method to determine γ from measurements of $\mathcal{A}_{\rm CP}^{mix}(B_d \to \pi\pi)$ and $\mathcal{A}_{\rm CP}^{dir}(B_d \to \pi\pi)$, to test the mechanism of CP violation in the SM and the possible existence of New Physics:
  - minimizing QCD theoretical input, using flavour symmetries of strong interactions. Sensitivity to U-spin breaking (ξ, $\Delta\theta$) has been explored.
  - control penguins using CP-averag. $\mathrm{BR}(B_d \to \pi^{\pm}K^{\mp})$.
- Together with R_b obtained from $b \to u/cl\bar{\nu}_l$ and $\sin\phi_d$ from CP-asymmetry of $B_d \to J/\Psi K_S$ we can fix the APEX of the UT, and obtain α and β:
  - Also in presence of New Physics affecting the mixing but not the $\Delta B = 1$, $\Delta S = 1$ amplitudes.

SCENARIO A: $\phi_d = 47^\circ$, data favours $\gamma < 90^\circ$.$\phi_d = 2\beta$ and γ agree with fits of UT in the SM $\Rightarrow \cos(\phi_d) > 0$.

Case of $\xi = 1$	α	β	γ
$\phi_d = 47^\circ$	$(103 \pm 29)^\circ$	$(20 \pm 7)^\circ$	$(57 \pm 22)^\circ$

SCENARIO B: $\phi_d = 133^\circ$, data favours $\gamma > 90^\circ$, CP-violating New Physics is required, large contribution to $\phi_d \Rightarrow \cos(\phi_d) < 0$.

Case of $\xi = 1$	α	β	γ
$\phi_d = 133^\circ$	$(44 \pm 20)^\circ$	$(15 \pm 7)^\circ$	$(123 \pm 22)^\circ$

- $B_s \to K^+K^-$: we obtain a *very constrained* allowed region for the SM, narrow target range for run II of Tevatron.
  - Combined with $B_d \to \pi\pi$ offers a **solid strategy** to measure γ (spectator quark hypothesis unnecessary).
  - New Physics contributions to $B_s - \bar{B}_s^0$ mixing may shift the range for the $\mathcal{A}_{\rm CP}^{mix}(B_s \to KK)$ significantly.