Bulk Fields with Brane Terms

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1. Brane Terms for Bulk Fields
2. Brane Kinetic Terms and Field Localization in Infinite Extra Dimensions
3. Phenomenology of Brane Kinetic Terms in Compact Extra Dimensions
   - Gauge Fields
   - Graviton
   - Fermions
4. General Brane Kinetic Terms
5. Conclusions

Brane Terms for Bulk Fields

Theories in extra dimensions:

- Fields propagating only on the defects (brane fields) + brane interactions
- Fields propagating in the bulk (bulk fields) + bulk interactions

+

- Bulk Field – Brane Field interactions
- Brane Terms for Bulk Fields. In particular, Brane Kinetic Terms (BKT)

Example:

$$S = \int \mathrm{d}^4x \int \mathrm{d}y \left\{ \partial_M \phi^\dagger \partial^M \phi + a\delta(y) \partial_\mu \phi^\dagger \partial^\mu \phi + \cdots \right\}$$

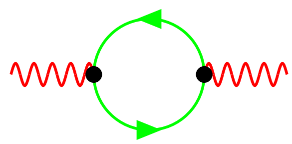
These brane terms are relevant to phenomenology and model building. We shall concentrate on BKT.

In generic models, the brane terms for bulk fields are necessary for the quantum consistency of the theory.

Indeed,

- Dvali, Gabadadze, Porrati, PLB 485 ('00) 208

  Bulk Field – Brane Field interactions

  ⇓ 

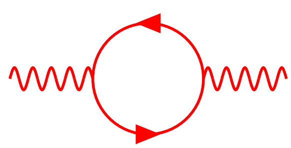
  Brane Localized (logarithmic) Divergencies.

  ⇓

  Brane Localized Counterterms
  (running of Brane Couplings)

- Georgi, Grant, Hailu, PLB 506 ('01) 207

  In Orbifolds, same effect even without brane fields or tree level brane couplings!



Size of brane couplings?

- The coefficients of the brane terms are in principle independent parameters to be determined by experiments.
- It is possible and natural to include them at tree level from the very begining $\Rightarrow$ no loop suppression.
- The brane terms are higher order in the low energy expansion of the effective theory $\Rightarrow$ extra $\frac{1}{\Lambda}$ suppression.
- Naive Dimensional Analisis $\rightsquigarrow$possible enhancement by geometrical factors.

Small BKT ($a << R$) can be important:

- Broken KK degeneracies

  Cheng, Matchev, Schmaltz, PRD 66 ('02) 056006
- Certain small BKT give rise to dramatic effects (wait and see)

Here, no *a priori* assumptions on the size of BKT

Quasilocalization in infinite extra dimensions

Dvali, Gabadadze, Porrati $\rightsquigarrow$ gravity

Dvali, Gabadadze, Shifman, PLB 497 ('01) 271 $\rightsquigarrow$ gauge bosons

$$\mathcal{L} = -\frac{1}{4g^2}\left(F_{MN}^2 + a\delta(y)F_{\mu\nu}^2\right) + \cdots$$

Modified (brane to brane) propagator:
(coordinate-momentum representation)

$$D_{\mu\nu}(p,0,0) = \frac{\eta_{\mu\nu}}{p^2 + 2p/a}[1 + \mathcal{O}(p)]$$

5D $\longrightarrow$ 4D transition
at $p \sim 2/a$

Coulomb's law:

$$V \sim \begin{cases} \frac{1}{r}, & r << a/2 \\ \frac{1}{r^2}, & r >> a/2 \end{cases}$$

"infrared transparency"

Phenomenology of BKT in compact extra dimensions

- Gauge bosons
  - Phenomenology of BKT
    - Flat space

      Carena, Tait, Wagner, Acta Phys.Polon.B33 ('02) 2355

    - Warped space

      Davoudiasl, Hewett, Rizzo, hep-ph/0212279
      Carena, Ponton, Tait, Wagner, PRD 67 ('03) 096006

  - Impact on model building
    - GUTs Hall, Nomura, PRD 66 ('02) 075004
    - Higgs as a gauge boson

      Scrucca, Serone, Silvestrini, hep-ph/0304220

- Graviton
  - Warped space

    Davoudiasl, Hewett, Rizzo, hep-ph/0305086

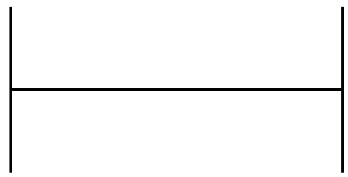
- Fermions
  - Flat space

    del Aguila, Pérez-Victoria, Santiago, hep-ph/0305119

Gauge bosons

Flat space

S^1/Z_2



$$\mathcal{L} = -\frac{1}{4g^2}\left[F_{MN}^2 + \left(a_0\delta_0 + a_\pi\delta_\pi\right)F_{\mu\nu}^2\right] + \cdots$$

KK decomposition (Gauge $A_5 = 0$)

$$A_\mu(x,y) = \sum_{n=0}^{\infty} \frac{f_n(y)}{\sqrt{2\pi R}} A_\mu^{(n)}(x)$$

with

$$\partial_y^2 f_n = -m_n^2 \left(1 + a_0\delta_0 + a_\pi\delta_\pi\right) f_n$$

$$\frac{1}{2\pi R}\int_{-\pi R}^{\pi R} \mathrm{d}y \left(1 + a_0\delta_0 + a_\pi\delta_\pi\right) f_n^2 = 1$$

- Flat zero mode

$$f_0 = \frac{1}{\sqrt{1 + \frac{a_0 + a_\pi}{2\pi R}}}$$

$$\boxed{a_0 + a_\pi \leq -2\pi R \Rightarrow \text{ghost}}$$

- Massive KK modes

$$f_n = A_n[\cos(m_n y) - \frac{a_0 m_n}{2}\sin(m_n y)]$$

$$(a_0 a_\pi m_n^2 - 4)\tan(\pi R m_n) - 2(a_0 + a_\pi)m_n = 0$$

$$\boxed{-2\pi R < a_{0,\pi} < 0 \Rightarrow \text{tachyon(s)}}$$

- Couplings to brane fields

$$\frac{1}{g_0^2} = \frac{2\pi R + a_0 + a_\pi}{g^2}$$

$$\frac{1}{g_n^2} = \frac{1/f_n(0)^2 + a_0 + a_\pi}{g^2}$$

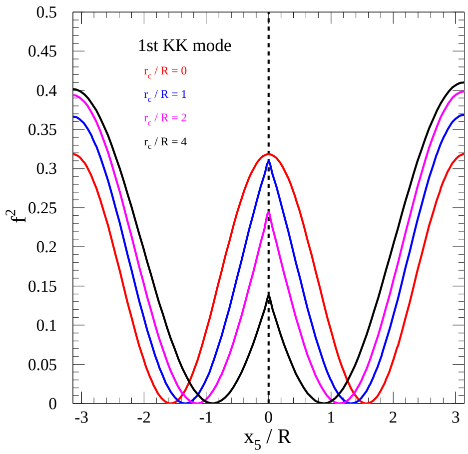
Light first mode (for $a_{0,\pi} \gg R$)

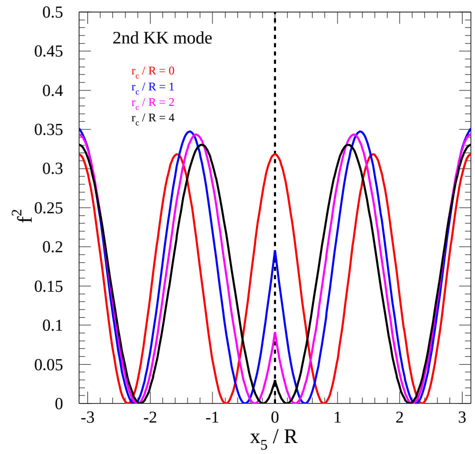
$$\boxed{m_1^2 \sim 2\frac{a_0 + a_\pi}{a_0 a_\pi \pi R}}$$

with same coupling to the branes as the zero mode for $a_0 = a_\pi \gg R$

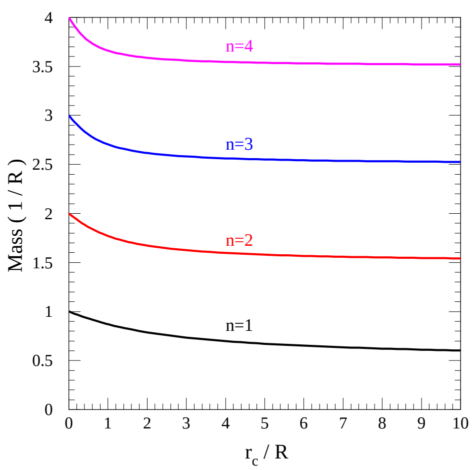
One opaque brane $(a_\pi = 0)$

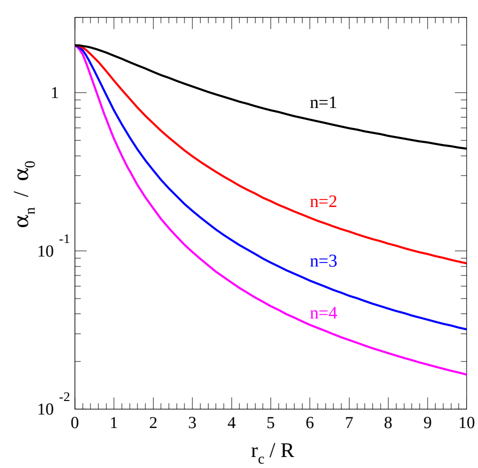
KK wave functions:





KK masses and couplings to the brane:



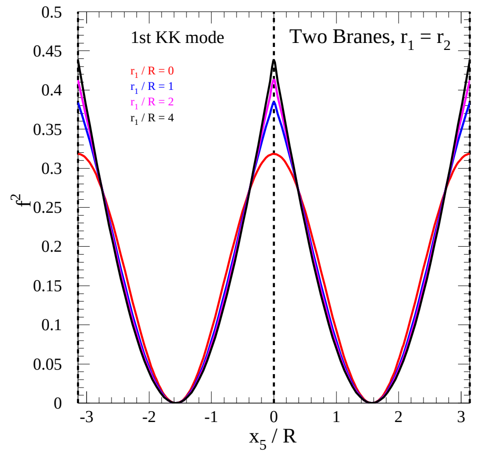


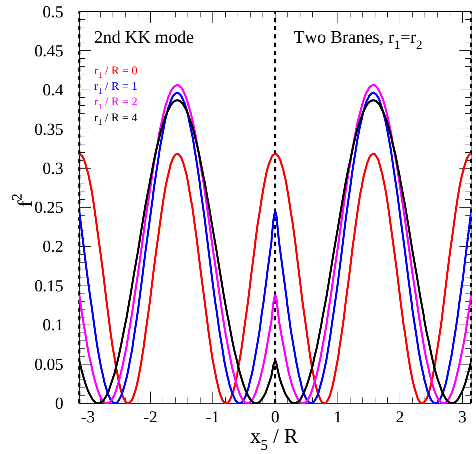
Brane "Opacity"

from Carena, Tait, Wagner

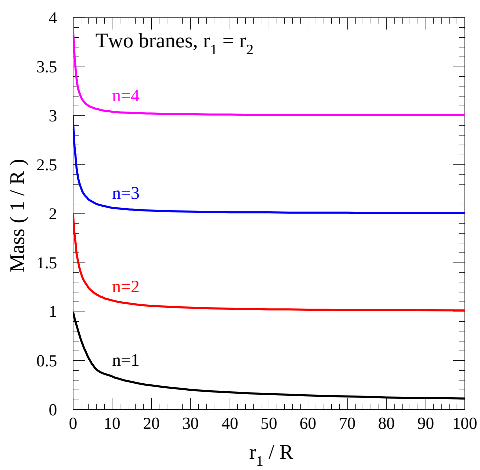
Two opaque branes (Symmetric case $a_0 = a_\pi$)

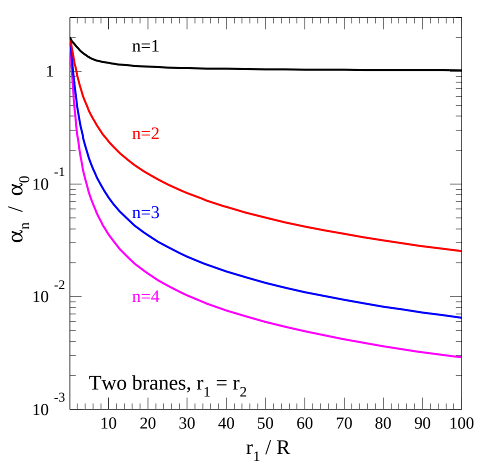
KK wave functions:





KK masses and couplings to the brane:

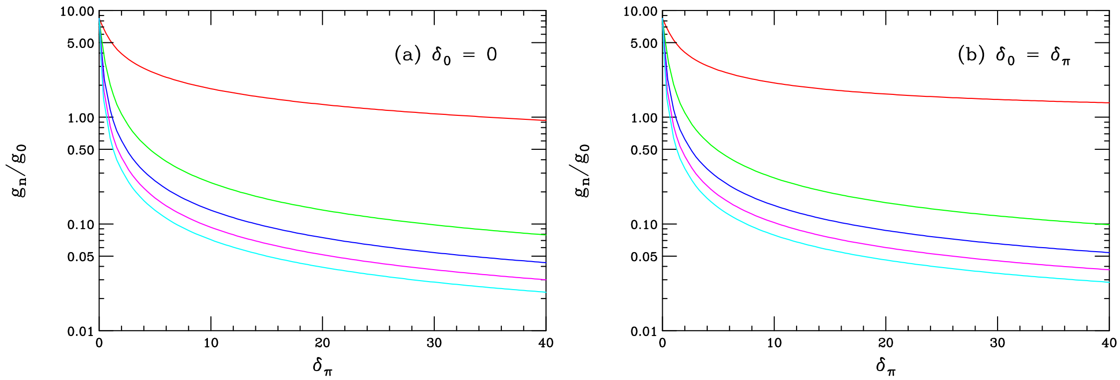




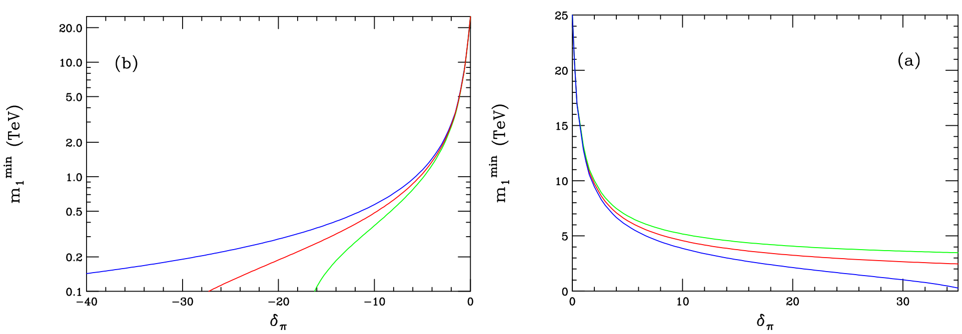
from Carena, Tait, Wagner

Randall-Sundrum $(\delta_I = \frac{1}{2} a_I k)$

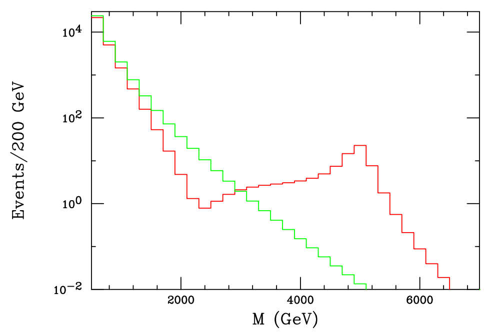
Couplings to electroweak brane



Lower bound on 1st KK mode from precision data



Drell-Yan production of 1st KK mode at LHC



from | Davoudias, Hewett, Rizzo |

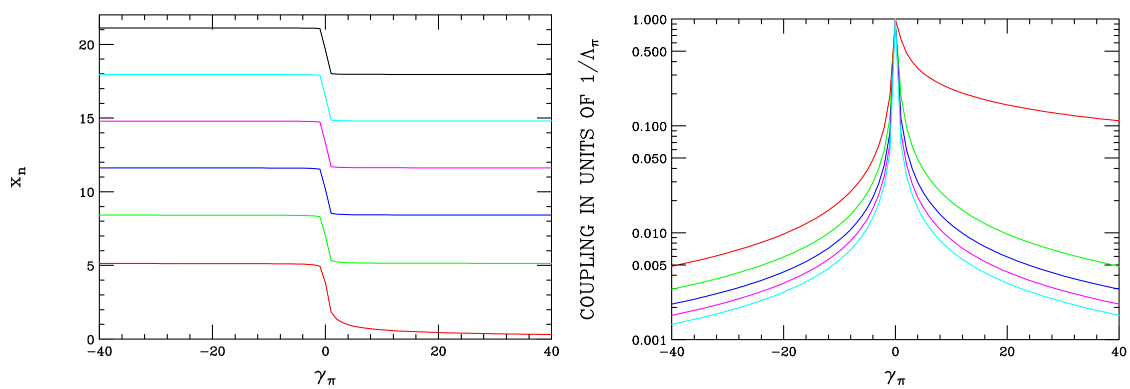
Phenomenology of BKT for gauge fields

- Opacity on one brane ⇒ weaker bounds on $1/R$ from virtual KK exchange in $e^+e^- \to \bar{f}f$ (fermions on opaque brane)
- Two opaque branes
  - Light mode could be discovered (large BKT)
  - Different KK couplings to quarks and leptons (living on separate branes)
  - Improve bounds on split fermion scenarios
- Randall-Sundrum with bulk gauge bosons more favorable:
  - $\Lambda_{\text{weak}} \sim 10$ TeV
  - Light gauge KK mode visible at LHC
- Grand Unification can be spoiled if BKT at the brane where the unified group is broken are not small

Graviton in Randall-Sundrum

$$S = \frac{M_5^3}{4} \int \mathrm{d}^4 x \int \mathrm{d}y \sqrt{-G} \{ R^{(5)} + (a_0 \delta_0 + a_\pi \delta_\pi) R^{(4)} \}$$

Reduced masses and couplings at the electroweak brane:



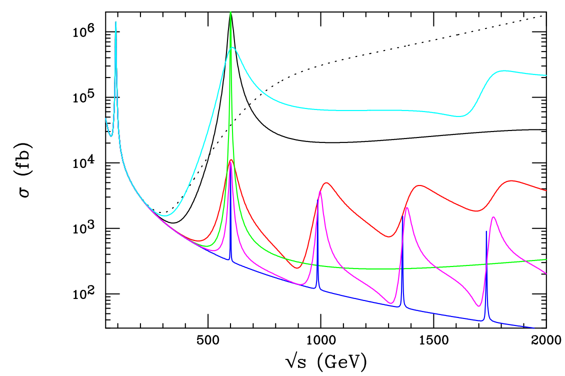
from Davoudias, Hewett, Rizzo

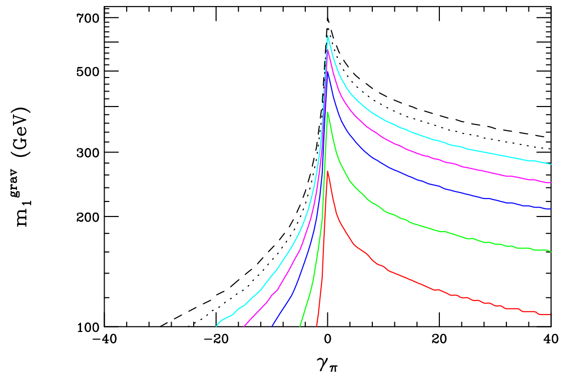
- 4D reduced Planck mass $\bar{M}_{Pl}^2 = \frac{M_5^3}{k}(1 + 2\gamma_0)$, $\gamma_I \equiv \frac{1}{2} a_I k$
- Effect of a_0 exponentially small at electroweak brane
- Light mode when $\gamma_\pi >> 1$

Cross section for $e^+e^- \to \mu^+\mu^-$

$m_1 = 600GeV, k/\bar{M}_{Pl} = 1, \gamma_\pi = 1, 2, -1, 25, -2, -10$

↓



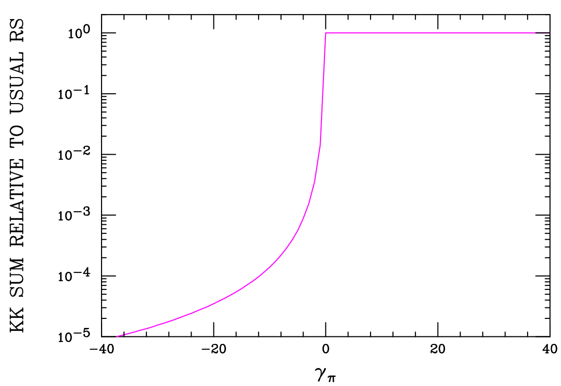


↑

Search reach for first KK resonance, Drell-Yan at LHC

$k/\bar{M}_{Pl} = 0.01, 0.025, 0.05, 0.075, 0.10, 0.125, 0.15$ upwards

Indirect effects ⇝Sum over weighted couplings:



Fermions

$$\mathcal{L} = \left[1 + a_0\delta_0 + a_\pi\delta_\pi\right]\bar{\Psi}_L \mathrm{i}\,\partial\!\!\!/\Psi_L$$
$$+ \left[1 + \tilde{a}_0\delta_0 + \tilde{a}_\pi\delta_\pi\right]\bar{\Psi}_R \mathrm{i}\,\partial\!\!\!/\Psi_R$$
$$- \bar{\Psi}_L\partial_y\Psi_R + \bar{\Psi}_R\partial_y\Psi_L$$

KK expansion

$$\psi_{L,R}(x,y) = \sum_n \frac{f_n^{L,R}(y)}{\sqrt{2\pi R}}\psi_{L,R}^{(n)}(x)$$

- Eigenvalue equation

$$\partial_y f_n^R = m_n[1 + a_0\delta_0 + a_\pi\delta_\pi] f_n^L$$
$$\partial_y f_n^L = - m_n f_n^R$$

- Normalization

$$\int \mathrm{d}y\ [1 + a_0\delta_0 + a_\pi\delta_\pi]\frac{f_n^L f_m^L}{2\pi R} = \delta_{nm}$$
$$\int \mathrm{d}y\ \frac{f_n^R f_m^R}{2\pi R} = \delta_{nm}$$

- Spectrum
  - Chiral massless zero mode

$$f_0^L = \frac{1}{\sqrt{1 + \frac{a_0 + a_\pi}{2\pi R}}}$$

  - Vector-like massive KK modes

$$f_n^L = A_n[\cos(m_n y) - \frac{a_0 m_n}{2} \sin(m_n y)]$$
$$f_n^R = - \partial_y f_n^L / m_n$$

with masses

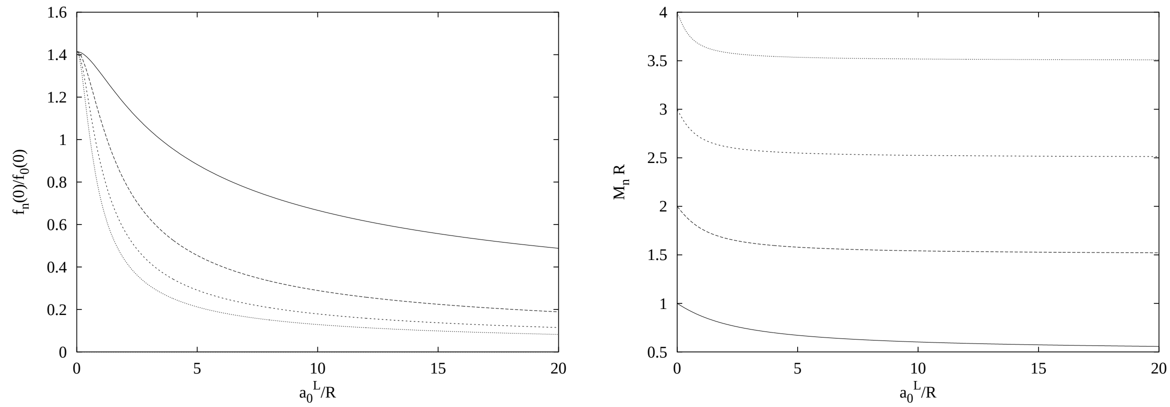
$$(4 - a_0 a_\pi m_n^2) \tan(m_n \pi R) + 2(a_0 + a_\pi) m_n = 0$$

Light first mode (for $a_{0,\pi} \gg R$)

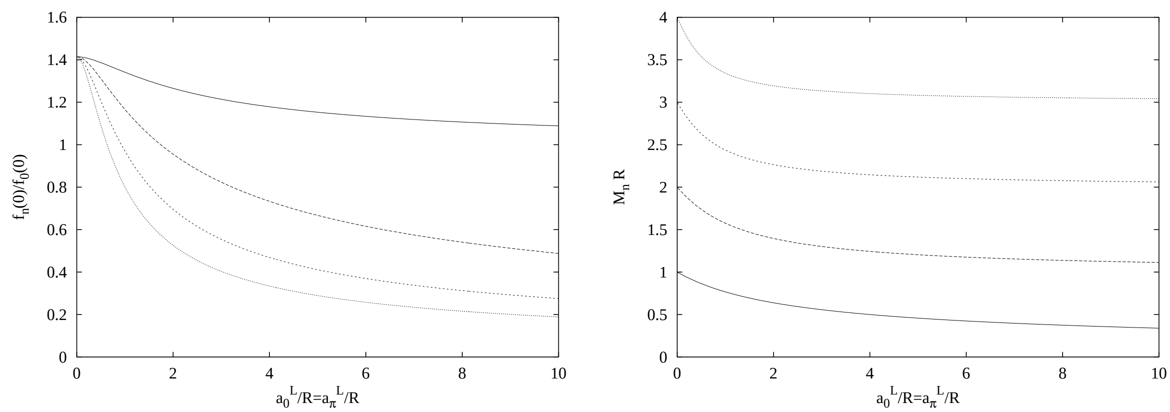
$$m_1^2 \sim 2 \frac{a_0 + a_\pi}{a_0 a_\pi \pi R}$$

with same coupling to the branes as the zero mode for $a_0 = a_\pi \gg R$ but decoupling in the one brane case

KK masses and couplings to the brane, $a_\pi = 0$



KK masses and couplings to the brane, $a_\pi = a_0$



Phenomenology: gauge interactions

$$\mathcal{L} = \int \mathrm{d}y \, (1 + a_I \delta_I) \bar{\Psi}_L \mathrm{i} \gamma^\mu D_\mu \Psi_L + \ldots$$

$$g^{(mnr)} = \frac{g_5}{\sqrt{2\pi R}} \int \mathrm{d}y \, (1 + a_I \delta_I) \frac{f_m^L f_n^L f_r^A}{2\pi R}$$

Gauge boson KK modes can be integrated out giving rise to four-fermion interactions with coefficient

$$V = m_W^2 \sum_{n>0} \frac{(g^{(00n)}/g^{(000)})^2}{m_n^2}$$

$$\frac{g^{(00n)}}{g^{(000)}} = \frac{a_0 f_n^A(0) + a_\pi f_n^A(\pi R)}{2\pi R + a_0 + a_\pi}$$

Experimentally

$$V \lesssim \begin{cases} 4.5 \times 10^{-3}, & \text{LEP}, \\ 1.2 \times 10^{-4}, & \text{NLC} \end{cases}$$

Forbidden regions from LEP (dark) and NLC (light), $a_\pi = 0$

Forbidden regions from LEP (dark) and NLC (light), $a_\pi = a_\pi$

Phenomenology: Yukawa couplings

- Consider boundary Yukawas (SUSY,…)
- Fermion mixing is proportional to fermion masses ⇒relevant for the top

$$W_{tb}^L \approx 1 - \frac{1}{2} m_t^2 \sum_n \left(\frac{f_n^{t_R}(0)}{f_0^{t_R}(0) m_n} \right)^2$$

- Contributions to T from mixing (no BKT)

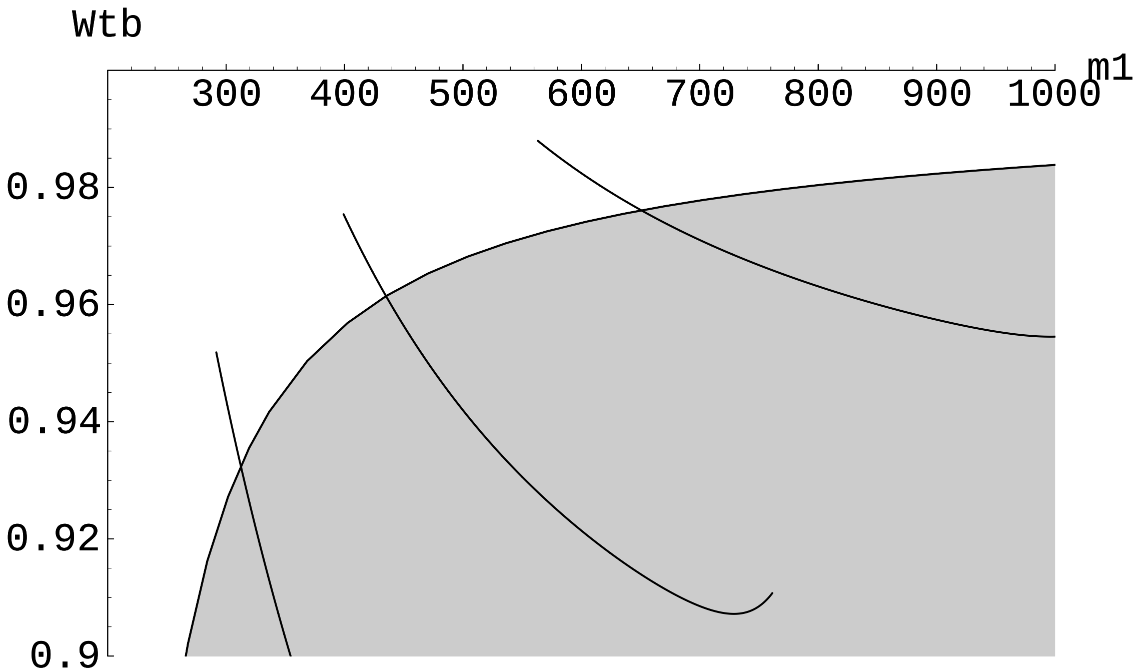
$$M_c \gtrsim 2.5 \text{ TeV } (W_{tb}^L = 0.992)$$

- BKT *save* the model in the one brane case
  - Decoupling from the brane and slightly lighter first mode
- Do not *save* the model in the $a_0 \sim a_\pi$ case
  - $f_1(0)/f_0(0) \sim 1$ but much lighter m_1

Value of CC top coupling (shaded region forbidden by T) for BKT and Yukawa at the same brane

$M_c = 500, 700, 1000$ GeV from left to right;

$0 \leq a_0 \leq 20R$.



General BKT

del Aguila, Pérez-Victoria, Santiago, JHEP 02 ('03) 051

- Scalars

$$\mathcal{L} = [1 + a_I\delta_I]\partial_\mu\phi^\dagger\partial^\mu\phi - [1 + c_I\delta_I]\partial_y\phi^\dagger\partial_y\phi + \frac{b_I}{2}\delta_I(\phi^\dagger\partial_y^2\phi + \partial_y^2\phi^\dagger\phi)$$

- Gauge bosons

$$\mathcal{L} = -\frac{1}{4}(1 + a_I\delta_I)F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(1 + c_I\delta_I)F_{5\nu}F^{5\nu}$$

- Fermions

$$\mathcal{L} = [1 + a_I^L\delta_I]\bar{\Psi}_L \mathrm{i}\,\partial\!\!\!/\Psi_L + [1 + a_I^R\delta_I]\bar{\Psi}_R \mathrm{i}\,\partial\!\!\!/\Psi_R - [1 + b_I\delta_I]\bar{\Psi}_L\partial_y\Psi_R - b_I\delta_I(\partial_y\bar{\Psi}_R)\Psi_L + [1 + c_I\delta_I]\bar{\Psi}_R\partial_y\Psi_L + c_I\delta_I(\partial_y\bar{\Psi}_L)\Psi_R$$

Singular behaviour (scalars) $a_\pi = b_\pi = c_\pi = 0$

Eigenvalue problem (regularization required)

$$[1+(b+c)\delta_0]\partial_y^2+(b+c)\delta_0'\partial_y+\frac{b}{2}\delta_0'' = -m_n^2\left(1+a\delta_0\right)f_n$$

- $b = 0$

$$f_n(y) = A_n[\cos(m_n y) - \frac{a m_n}{2}\sin(m_n|y|)]$$

$$\tan(\pi R m_n) + \frac{a}{2}m_n = 0.$$

- $b \neq 0$

$$f_n(y) = A_n \sin(m_n|y|)]$$

$$m_n = \frac{n+1/2}{R}$$ No zero mode!

Independent of brane couplings!

Dramatic changes for arbitrarily small b

$\downarrow$

Breakdown of the low-energy expansion.

(Classical) Renormalization

This singular behaviour comes from the zero width brane limit and is related to the appearance of $\delta(0)$'s in perturbation theory.

The singularities can be cancelled by (higher order) counterterms.

At second order,

$$\mathcal{L}_{\rm ct} = \frac{b^2}{4}\Big\{ - \delta_0^2(y)\partial_\mu\phi^\dagger\partial^\mu\phi - \partial_y\left[\delta_0(y)\phi^\dagger\right]\partial_y\left[\delta_0(y)\phi\right] + \delta_0^2(y)\left(\phi^\dagger\partial_y^2\phi + \partial_y^2\phi^\dagger\phi\right)\Big\}$$

Renormalized masses:

$$\tan(\pi R m_n) - \frac{b}{2}m_n = 0$$

Smooth in b!

Conclusions

- BKT present in theories extra dimensions with defects
- Independent parameters
  - Interesting impact on phenomenology
  - Must be taken into account in model building
- Some BKT give rise to singular behaviour, which can be smoothed down via (classical) renormalization
- Open questions:
  - Systematic renormalization (w/ quantum corrections)
  - General BKT in gravity
  - Interplay with other Brane Terms
  - New phenomenological effects